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The collision of particles in granular systems

Nikolai V. Brilliantov[a,*], Frank Spahn[b], Jan-Martin Hertzsch[b], Thorsten Pöschel[c]

[a] *Moscow State University, Physics Department, Moscow 119899, Russia*
[b] *University Potsdam, Am Neuen Palais, D-14415 Potsdam, Germany*
[c] *Humboldt-Universität zu Berlin, Institut für Physik, Invalidenstraße 110, D-10115 Berlin, Germany*

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Abstract

Collisions between granular particles are irreversible processes which cause dissipation of mechanical energy by fragmentation or heating of the colliders. The knowledge of these phenomena is essential for the understanding of the behaviour of complex systems of granular particles. We have developed a model for inelastic collisions of granular particles and calculated the velocity restitution coefficients, which describe all possible collisions in the system. The knowledge of these coefficients allows for event-driven many-particle simulations which cannot be performed in the frame of molecular dynamics. This approach has the advantage that very large particle numbers can be treated which are necessary for the understanding of intrinsic large-scale phenomena in granular systems.

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Systems of granular particles are very common in nature, from terrestrial sands and gravels to the dust and planetary rings in space. These systems are also of importance in modern technology. Striking properties of the granular materials, which exhibit a solid-like behavior in unperturbed states and fluid-like behavior in perturbed states caused much interest in these systems in the past few years. The numerical studies of the granular materials carried out recently (e.g. [1]) were based on the molecular dynamics technique, where all forces and moments acting between each pair of colliding particles must be calculated explicitly (e.g. [2]). These simulations are extremely time consuming, and thus only granular systems with a relatively small number of particles could be studied so far.

* Corresponding author. E-mail: nbrillia@alchemy.chem.utoronto.ca.

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In order to understand phenomena such as the origin of structures in planetary rings (e.g. [3]) one has to simulate much larger systems than it would be possible by using molecular dynamics.

Alternative simulation methods are based on "event driven" algorithms (EDA). Using an EDA the computer time is proportional to the number of events, i.e. collisions. The simple propagation of particles in the time between collisions does not require computational effort. In EDA, the collisions are not considered in detail by numerically integrating repulsive forces during a collision where each collision needs tens or hundreds of time steps and hence force evaluations. Instead one assumes the idealization that the particles are hard and therefore there are only pairwise interactions and the duration of a collision is assumed to be zero. These idealizations are well justified for the case of dense granular systems where the relative velocities are small.

For molecular dynamics one has to provide the proper expressions for the interaction forces. Similarly, for EDA, one needs the transformation rules for the relative velocities after a collision as a function of the impact velocities. The aim of the present paper is to derive these rules for granular gases from the viscoelastic properties of the grain material.

We describe the collision of grains – modelled as rough spheres – using the concept of restitution coefficients. This relates the relative velocities of a pair of particles after the collision $(v_{N;T})'$ to those before the impact without considering the details of the collision [4]:

$$(v_N)' = -\varepsilon_N v_N \quad (0 \leqslant \varepsilon_N \leqslant 1), \tag{1a}$$

$$(v_T)' = \varepsilon_T v_T \quad (-1 \leqslant \varepsilon_T \leqslant 1). \tag{1b}$$

The indices N and T label the normal and tangential components with respect to the contact plane. The term "impact velocity" means the relative velocity of the surfaces of the bodies at the point of contact. Its components are functions of the relative translational and rotational velocities of the particles and of their radii (e.g. [5]). The normal impact velocity $\boldsymbol{v}_N$ is in parallel with the normal unit vector $\boldsymbol{n}$ of the contacting surfaces, whilst the tangential impact velocity $\boldsymbol{v}_T$ is in perpendicular to $\boldsymbol{n}$. Thus, the investigation of dynamics of the system reduces to the determination of the functions $\varepsilon_N(v_N, v_T)$ and $\varepsilon_T(v_N, v_T)$, which describe entirely the collisions and which will be discussed in the following. Knowing the functions $\varepsilon_N(v_N, v_T)$ and $\varepsilon_T(v_N, v_T)$ one can set up an event driven simulation which is much less time consuming than molecular dynamics. A very effective implementation of such an algorithm was described by Rapaport [6].

Experimental results (e.g. [7,8]) show that the coefficients of restitution depend sensitively on the impact velocity. Furthermore, these coefficients determine further evolution of the velocities $\boldsymbol{v}$ and angular velocities $\boldsymbol{\omega}$.

In the framework of visco-elasticity we derive ordinary differential equations for the compression $\xi = R_i + R_j - (|\, \boldsymbol{r}_i(t) - \boldsymbol{r}_j(t)\, |)$ (in normal direction $\boldsymbol{n}$) and the tangential

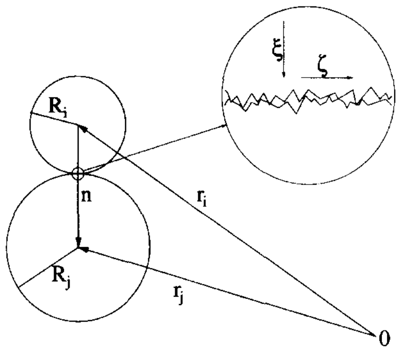

Fig. 1. Sketch of two colliding spheres.

deformation ζ (direction perpendicular to $\boldsymbol{n}$) of both spheres in contact [4]. For the definition of the symbols used we refer to Fig. 1.

In the case of elastic deformation the Hertz contact law [9]

$$F^{\mathrm{N}}_{(\mathrm{el})}(\xi) = \frac{2Y}{3(1-\nu^2)}\sqrt{R_{\mathrm{eff}}}\;\xi^{3/2} \tag{2}$$

describes the restoring force. $R_{\mathrm{eff}} = R_i R_j/(R_i + R_j)$, Y, and ν are the effective radius, the Young modulus, and the Poisson ratio, respectively. Under quasistatic conditions where the impact velocity is small compared with the speed of sound in the material, the dissipative part of the normal force can be written as [4]

$$F^{\mathrm{N}}_{(\mathrm{dis})} = \frac{Y}{(1-\nu^2)}\sqrt{R_{\mathrm{eff}}}\;A\sqrt{\xi}\,\dot{\xi}. \tag{3}$$

Then, the normal component of relative motion is described by the equation

$$\ddot{\xi} = -B\left(\xi^{3/2} + \frac{3}{2}\,A\sqrt{\xi}\,\dot{\xi}\right) \approx -B(\xi + A\,\dot{\xi})^{3/2} \tag{4}$$

with the initial conditions $\dot{\xi}(0) = v_{\mathrm{N}}$ and $\xi(0) = 0$ (relative velocity and penetration depth at the beginning of the collision). A and B are well defined material constants (see [4]):

$$A = \frac{1}{3}\frac{(3\eta_2 - \eta_1)^2}{(3\eta_2 + 2\eta_1)}\left(\frac{(1-\nu^2)(1-2\nu)}{Y\nu^2}\right), \qquad B = \frac{2Y\sqrt{R^{\mathrm{eff}}}}{3m^{\mathrm{eff}}(1-\nu^2)}, \tag{5a, b}$$

where $\eta_{1/2}$ being the elastic and the viscous constants of the particle material. Surprisingly, the approximate part of Eq. (4) has a similar form as the Hertz law (2) if the condition $A\dot{\xi} \ll \xi$ holds, which is the case for the entire collision process. With the solution of Eq. (4) the restitution coefficient reads

$$\varepsilon_{\mathrm{N}} = -\dot{\xi}(t_{\mathrm{c}})/\dot{\xi}(0), \tag{6}$$

where t_{c} is the duration of the collision.

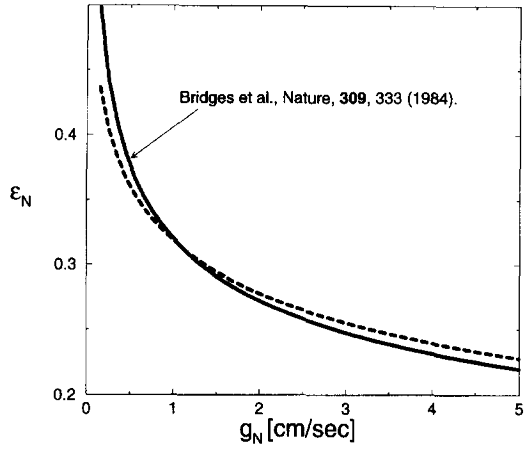

Fig. 2. The theoretical results for the normal restitution coefficient ε_N as a function of the normal component of the impact velocity g_N agree with the experimental results [7] (dashed line).

Fig. 2 shows the comparison of the restitution coefficient obtained by Eqs. (4)–(6) (for ice at low temperatures: $Y = 10\,\mathrm{GPa}$, $\nu = 0.3$, $R = 10^{-2}\,\mathrm{m}$, $\rho = 10^3\,\mathrm{kg\,m^{-3}}$ [10]), and the results of experiments by Bridges et al. [7]. The experimental data agree well with the theoretical results.

To derive the equation for the tangential motion one has to provide an expression for the tangential force F_T between the colliding particles. The rough surfaces of the bodies in contact are modelled by asperities which may vary in shape and size by several decades. (see e.g. [11]). The transmission of momentum is caused by the contact of the asperities with the mean size L. The relative tangential motion $\dot{\zeta} = v_T$ of the particle surfaces causes a shear deformation $\overline{\eta} \propto \zeta/L$ of the asperities, resulting in a tangential stress σ_T. The force $F_T = S_T\overline{\sigma_T}$ causing this stress is transmitted by the portion $S_T(t) = f_T(\overline{\sigma_N})S(t) \approx \phi_T\,\overline{\sigma_N}\,S(t)$ of the nominal contact area S. For small deformations ζ, the surface S_T is approximately a linear function of S and of the mean pressure $\overline{\sigma_N}$ exerted on the surface, and the shear results in almost elastic stresses.

When these stresses exceed some critical value σ_T^* which is a specific material constant, the asperity which hinders the tangential relative motion of the surfaces is assumed to break, i.e. to dissipate the energy which was stored in elastic stresses. This results in a sudden release of the shear stress, and the particle surfaces are shifted by a distance ζ_0 with respect to each other. This yields the shear deformation

$$\overline{\eta}(\zeta) = \eta_* \left(\frac{\zeta}{\zeta_0} - \left\lfloor \frac{\zeta}{\zeta_0} \right\rfloor \right), \tag{7}$$

where $\lfloor x \rfloor$ denotes the integer part of x and η_* is the maximal elastic shear deformation. Thus, one obtains the tangential force

$$F_T = -\mu F_N \left(\frac{\zeta}{\zeta_0} - \left\lfloor \frac{\zeta}{\zeta_0} \right\rfloor \right), \tag{8}$$

where $F_N = \overline{\sigma_N} S = -m^{\rm eff}\ddot{\xi}(t)$ and $\mu = \phi_T \eta_*$. Then, the maximum possible tangential force given by

$$F_T^{\max} = \mu F_N \tag{9}$$

reproduces the Coulomb friction law [12] and explains the friction coefficient μ as a function of mesoscopic properties of the surfaces, i.e. their roughness. The idea that the contact breaks by a permanent dissipative deformation of the asperities when the shear stress exceeds a critical value is closely related to the extensively investigated one-dimensional model of Burridge and Knopoff [13]. Summarizing our model covers both this case and Coulomb's law.

With the tangential force (8) we obtain the differential equation for the motion in shear direction

$$\ddot{\zeta} - \frac{\mu}{\kappa}\ddot{\xi}(t)\left(\frac{\zeta}{\zeta_0} - \left\lfloor\frac{\zeta}{\zeta_0}\right\rfloor\right) = 0 \tag{10}$$

with the initial conditions $\dot{\zeta}(0) = v_T$ and $\zeta(0) = 0$. The coefficient κ here is expressed in terms of masses and moments of inertia of colliding particles [4]. The tangential restitution coefficient can be calculated from

$$\varepsilon_T = \dot{\zeta}(t_c)/\dot{\zeta}(0). \tag{11}$$

Eq. (10) has been solved numerically for initial normal and tangential velocities in the range $|v_{N;T}| \leqslant 5$ cm/s, which is, for instance, appropriate for collisions in planetary rings. Figs. 3 and 4 show the tangential restitution coefficient ε_T as a function of the impact velocities for smooth (upper part) and rough spheres (lower part). A complex, nonlinear dependence of the tangential restitution coefficient on the impact velocity can be observed. In particular, a transition from sliding to rolling behaviour occurs if the asperities are not too small in comparison to the size of the colliding bodies. This behaviour which is due to the Coulomb friction law is very well known from every day experience.

In this paper we have presented an analytical model of collisions of granular grains which accounts for the energy dissipation in granular systems. The description of the normal motion is a generalization of Hertz theory [9]. The results are confirmed by a good agreement with experimental data. For the tangential coefficient we find that our description covers both the Coulomb friction and the widely used friction model by Burridge and Knopoff.

Furthermore, our model enables theoretical investigations of much larger systems than it would be possible by using molecular dynamics. Among the most interesting granular systems are planetary rings. It is known that they reveal complex structures in radial as well as in azimuthal direction whose origins are not sufficiently explained so far. Of particular interest are transport processes and the formation of dissipative structures in the rings. We hope to get further insight into the principles of planetary ring genesis from numerical results using an event driven algorithm with the transition rules derived in this paper. This work is in progress.

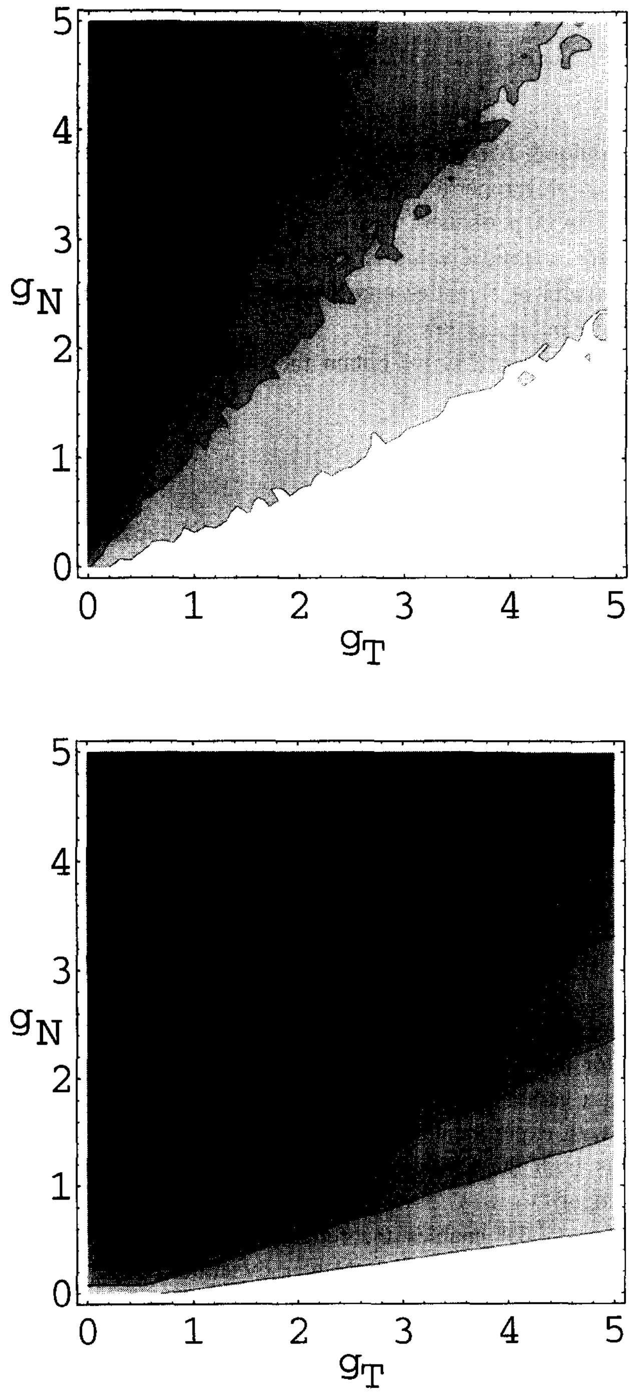

Fig. 3. The tangential restitution coefficient ε_T as function of the impact velocities g_N and g_T for two values of the surface roughness: (top) $\zeta_0 = 10^{-7}R^{\mathrm{eff}}$, (bottom) $\zeta_0 = 2 \times 10^{-4}R^{\mathrm{eff}}$.

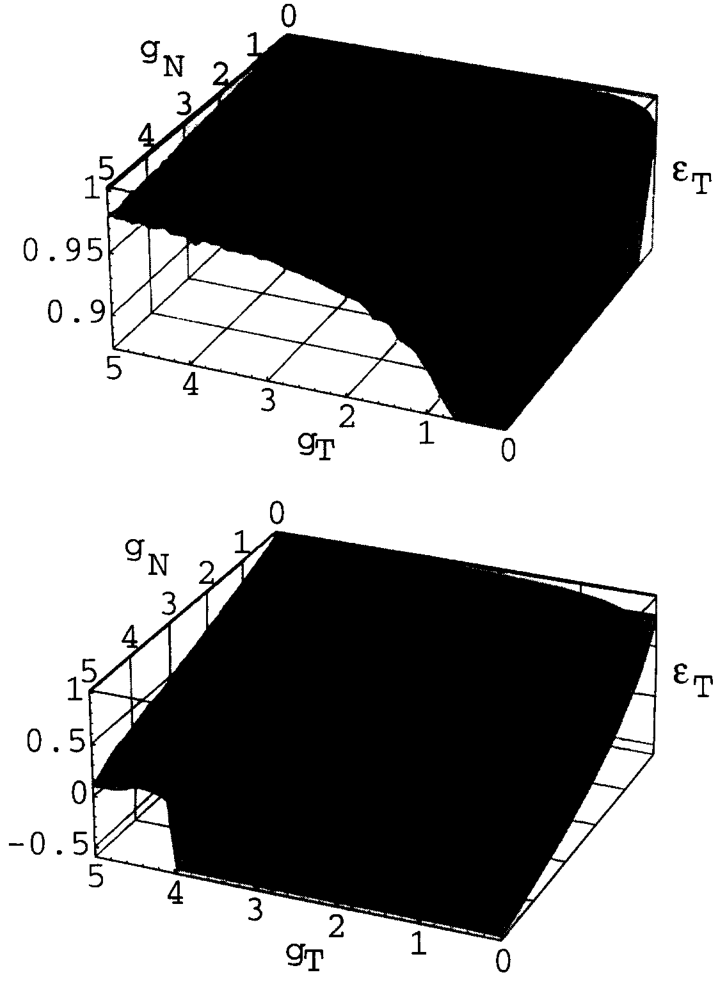

Fig. 4. Surface plot of the tangential restitution coefficient ε_T versus the plane g_N–g_T of the impact velocities for two values of the surface roughness (same values as in Fig. 3).

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