Dense packing of spheres in cylinders

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We study the optimal packing of hard spheres in an infinitely long cylinder. Our simulations have yielded dozens of periodic, mechanically stable, structures as the ratio of the cylinder (D) to sphere (d) diameter is varied [Mughal, 2011]. Up to D/d=2.715 the densest structures are composed entirely of spheres which are in contact with the cylinder. The density reaches a maximum at discrete values of D/d when a maximum number of contacts are established. These maximal contact packings are of the classic "phyllotactic" type, familiar in biology. However, between these points we observe another type of packing, termed line-slip.

An analytic understanding of these rigid structures follows by recourse to a yet simpler problem: the packing of disks on a cylinder. We show that maximal contact packings correspond to the perfect wrapping of a honeycomb arrangement of disks around a cylindrical tube. While line-slip packings are inhomogeneous deformations of the honeycomb lattice modified to wrap around the cylinder (and have fewer contacts per sphere).

Beyond D/d=2.715 the structures are more complex, since they incorporate internal spheres [Mughal, 2012], but an analysis in terms of contacts or constraints is still illuminating. We review some relevant experiments with hard spheres and small bubbles. We also discuss on-going and future areas of work related to this project



Mughal A., Chan H. K., and Weaire D., Physical Review Letters, 106, 115704, 2011. Mughal et al., Physical Review E, 85, 051305, 2012.