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Recurrent clogging and density waves in granular material flowing through a narrow pipe

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Abstract. — We report on density waves in granular material, investigated both experimentally and numerically. When granular material falls through a long narrow pipe one observes recurrent clogging. The kinetic energy of the falling particles increases up to a characteristic threshold corresponding to the onset of recurrent clogging and density waves of no definite wavelength. The distances between regions of high density depend strongly on the initial conditions. They vary irregularly without any characteristic time and length scale. The particle–flow was investigated using 2D Molecular Dynamics simulations. Experimental investigations lead to equivalent results.

The astonishing effects observed in moving granular material have been subjects of interest for at least 200 years. Examples of such effects are the angle of repose [1, 2] size segregation [3-5], heap formation on vibrating media [6-9], density waves [10] and granular flow down inclined surfaces [11, 12]. The origin of these effects is that granular material can behave like a solid or like a liquid, depending on its density [13]. Particularly, in recent times dynamic as well as static behaviour of granular material has been investigated by many authors experimentally and theoretically with various techniques either analytical, such as thermodynamic [14] and hydrodynamic [15] approaches, or numerical like cellular automata [16], Monte–Carlo simulation [4] and molecular dynamics [3, 8, 17]. Even random walk approaches have been proposed [18].

The aim of this Letter is to report the effect that granular material like sand that falls through a vertical narrow pipe or capillary with a diameter of only few particle-diameters changes from a homogeneous to an inhomogeneous flow when the kinetic energy E_k of the falling grains reaches a characteristic threshold. In this state one observes recurrent clogging of the particle flow (stick slip motion) and density waves. We simulated this behaviour using molecular dynamics and demonstrate that the results agree with our experimental observations.

The experiments were carried out in two vertical fixed glass–pipes of diameters $D_1 = 2$ mm and $D_2 = 4$ mm and length $L = 1.4$ m. An upper funnel contained enough granular material to ensure time–independent initial conditions. During the experiments very fine–grained sand of typical particle diameter $D_g = 0.18$ mm begins to flow homogeneously with the velocity $v = 0$ at the upper end of the pipe. After a typical distance of approximately 20 cm the uniform flow becomes unstable and turns into a flow of recurrent clogging. In this regime density waves can be observed. Figure 1 shows the pipes with typical density distributions. The distances between dark high–density regions vary irregularly. There is no definite wavelength. The distance between the top of the tubes and the onset of the stick slip motion as well as for the distances between the plugs depends sensitively on the humidity of the air and varies significantly from one experiment to the next one, i.e. depend on initial conditions. Similar experiments for the flow of sand through hourglasses have been described in [19]. As in the case of our numerical simulations they found clogging, the distribution of the distances between the high–density regions obeys a power law.

In fact we have done the simulations first and the reported experiments were intended to check the accuracy of the results predicted by the numerical simulations.

According to the experiments reported above we investigated numerically the flow of spherical-shaped grains with rotational degrees of freedom falling under gravity through a pipe of diameter D (Fig. 2) using 2D molecular dynamics simulations. The length of the pipe considered was much larger then its diameter $L \gg D$. We investigated systems of particles of equal radii $r_i = R$ as well as of particles with a Gaussian probability distribution for the radii with mean value R (Fig. 3). For the flow we assumed periodic boundary conditions. The inner surface of the pipe is build of smaller spheres with a diameter $r_s = \frac{2}{3} \cdot R$ of the same material as the grains to simulate a rough surface. Since our model includes elastic and inelastic interaction between the grains each other and between the grains and the wall we simulated the wall exactly in the same manner as the grains but the particles the pipe is build of were neither allowed to move nor to rotate.

The grains with mass m_i were accelerated due to gravity ($g = -9.81$ m/s^2, their density $\frac{3 \cdot m_i}{4 \cdot \pi \cdot r_i^3}$ is set to unity):

$$F_z = -m_i \cdot g \tag{1}$$

where F_z is the force acting upon the ith particle in the direction of the axis of the pipe.

Starting with given initial positions of the particles and randomly chosen small initial linear and angular velocities the dynamics of the system was determined by integrating Newtons equation of motion numerically for each particle for all further time steps. For the integration we used a sixth order predictor-corrector method for the particle positions and a fourth order for the calculation of the particles' angular movement [20].

In our model particles are allowed to penetrate each other slightly. The force acting between two particles i and j is assumed to be zero in case the particles do not touch each other, i.e. when the distance between their centres is larger then the sum of the particle radii $r_i + r_j$. Otherwise the force separates into normal and shear force (Eq. (2)).

$$\mathbf{F}_{ij} = \begin{cases} F_N \cdot \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|} + F_S \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|} & \text{if } |\mathbf{x}_i - \mathbf{x}_j| < r_i + r_j \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

with

$$F_N = k_N \cdot (r_i + r_j - |\mathbf{x}_i - \mathbf{x}_j|)^{1.5} - \gamma_N \cdot m_{\text{eff}}(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j) \tag{3}$$

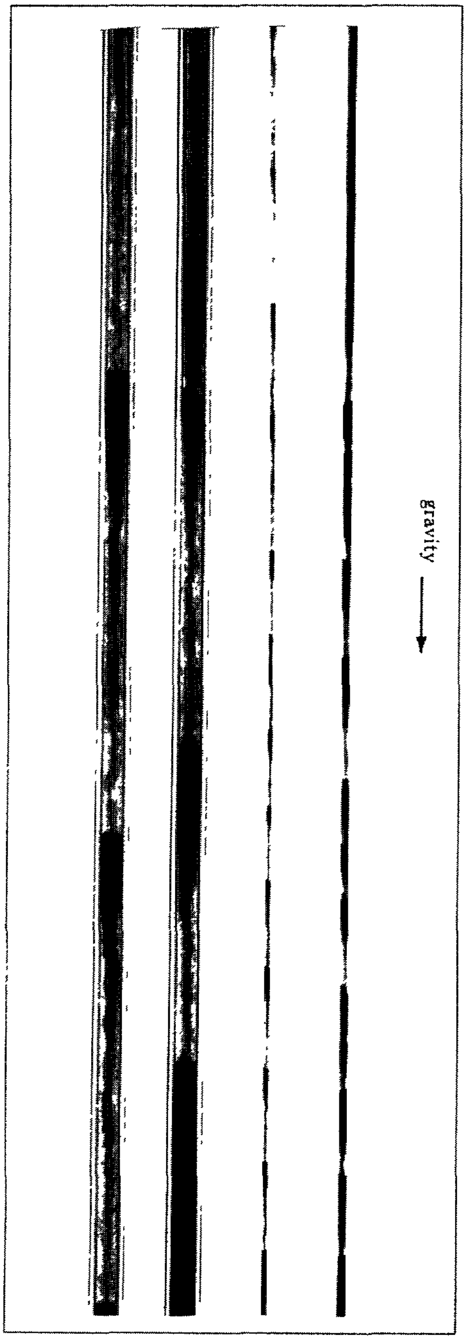

Fig. 1. — The experiment: fine grained sand with a typical particle diameter of $D_g = 0.18$ mm flows through a glass capillary tube. Recurrent clogging and density waves as well as free falling grains are visible. The distances between the (dark) regions of high density fluctuates significantly. The figure shows two pipes with different diameters (2 mm and 4 mm, length 1.4 m) and enlargements of the thinner one.

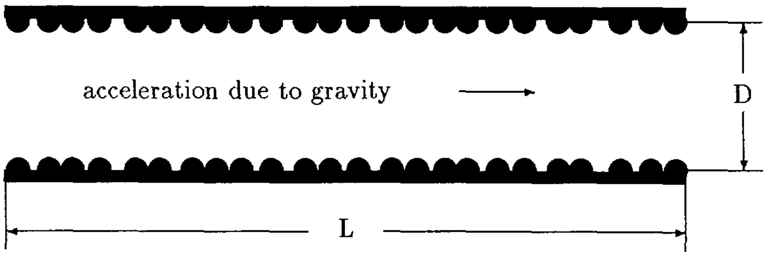

Fig. 2. — The pipe. The rough surface consists of spheres with diameter $r_s = \frac{2}{3} \cdot R$ of the same material as the grains.

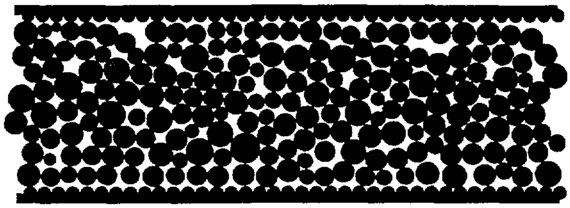

Fig. 3. — Enlargement of a small part of the pipe filled with grains of different radii. Gravity acts horizontally from left to right.

and

$$F_{\mathrm{S}} = \min\{-\gamma_{\mathrm{S}} \cdot m_{\mathrm{eff}} \cdot v_{\mathrm{rel}}\ ,\ \mu \cdot |F_{\mathrm{N}}|\} \tag{4}$$

where

$$v_{\mathrm{rel}} = (\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j) + R_i \cdot \dot{\Omega}_i + R_j \cdot \dot{\Omega}_j \tag{5}$$

$$m_{\mathrm{eff}} = \frac{m_i \cdot m_j}{m_i + m_j} \tag{6}$$

The terms $\mathbf{x}_i$, $\dot{\mathbf{x}}_i$ and $\dot{\Omega}_i$ stand for the coordinate, the velocity and the angular velocity of the ith particle. equation (3) includes the Hertzian contact force which rises with the power 1.5 of the penetration depth $r_i + r_j - |\mathbf{x}_i - \mathbf{x}_j|$ of two particles. It is used to mimic 3D behaviour of the grains [21]. This ansatz for the force was suggested in [22] and slightly modified in [3]. Equation 4 takes into account, that the maximum momentum two particles are able to transmit while colliding is determined by the Coulomb friction law [1]. μ is the Coulomb-coefficient of the particles.

To discuss the observed phenomena we refer to simulations with $N = 600$ particles. The length of the pipe was $L = 666 \cdot R$ and its width $D = 5 \cdot R$. The shear friction coefficient was $\gamma_{\mathrm{S}} = 3 \times 10^3\ \mathrm{s}^{-1}$, the normal friction coefficient $\gamma_{\mathrm{N}} = 3 \times 10^3\ \mathrm{s}^{-1}$, the material constant $k_{\mathrm{N}} = 10^5 \frac{\mathrm{N}}{\mathrm{m}^{1.5}}$ and $\mu = 0.5$. For the integration time step was chosen $\Delta t = 10^{-5}$ s. For this value the calculation is numerically exact, i.e. the results do not depend on Δt.

After filling the particles into the pipe they start moving at rest. While falling they gain kinetic energy due to the constant acceleration. To avoid infinitely rising velocity of the

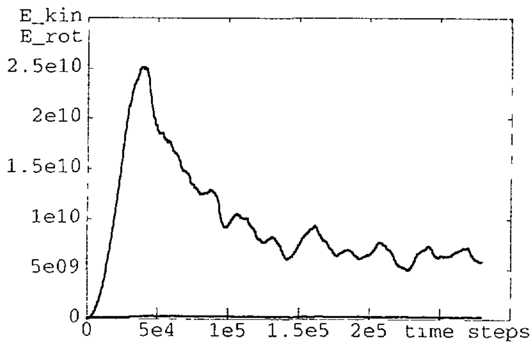

Fig. 4. — Evolution of the kinetic energy of the transversal particle movement and the rotational energy. Since the rotational energy is very small the curve is almost indistinguishable of the time-axis. While flowing accelerated, the system becomes unstable at 35.000 time steps approximately and transits into another flow regime where it reaches a state of relative steady kinetic energy ($E \approx 6 \times 10^9$ for this simulation).

particles without colliding they were initialized with a very small random velocity perpendicular to the axis of the pipe. At a certain energy the flow does not get faster anymore but the grains are decelerated very intensively. From now on the system moves with approximately constant kinetic energy. Figure 4 shows the evolution of the kinetic energy of the transversal particle movement E_{k} and the energy of the particle rotation E_{r}.

In the case of the low velocity regime the granular material moves through the pipe with almost homogeneous particle density. Figure 5 displays snapshots of the pipe each 1,000 time steps. Time increases upwards while horizontally one sees the evolution of the density wave from the left to the right. Gravity acts from left to right. In the early times the particle-density is approximately homogeneous due to the initial conditions. During the evolution it becomes more and more inhomogeneous and density waves are observed. As visible at later times the flow is unstable: the regions of high density can diverge as well as converge while the average velocity remains approximately the same as shown in figure 4.

After the transition into the recurrent-plug-flow regime, where the velocity profile does not vary with the distance from the centre on the pipe, the system reaches a dynamic state of nearly constant kinetic energy due to the equilibrium between constant acceleration and energy dissipation by friction. Nevertheless one can observe small fluctuations of the energy which are approximately due to the genesis and vanishing of regions of higher density with time, as visible comparing the evolution of the energy (Fig. 4) with the corresponding snapshot at the same time (Fig. 5). This behaviour proves that the inhomogeneous flow is unstable and it could be an indication of coexisting metastable states but we did not check this possibility yet. Our simulations show further that there are no significant differences between the behaviour of a system with particles of equal and randomly distributed radii. Simulating the system with slightly different initial conditions we got very similar results for the energy evolution and for the structure of the density wave but the concrete density wave as it can be observed as dark and bright regions in figure 5 varies significantly. Very small differences in the initial conditions lead to very different macroscopic behaviour. In this sense we call the flow chaotically.

There are essential differences between the behaviour of a liquid and a fluidized granular material: for the case of an incompressible fluid one never finds density waves. Nevertheless there are similarities too: The evolution of the energy of the system is similar to the evolution

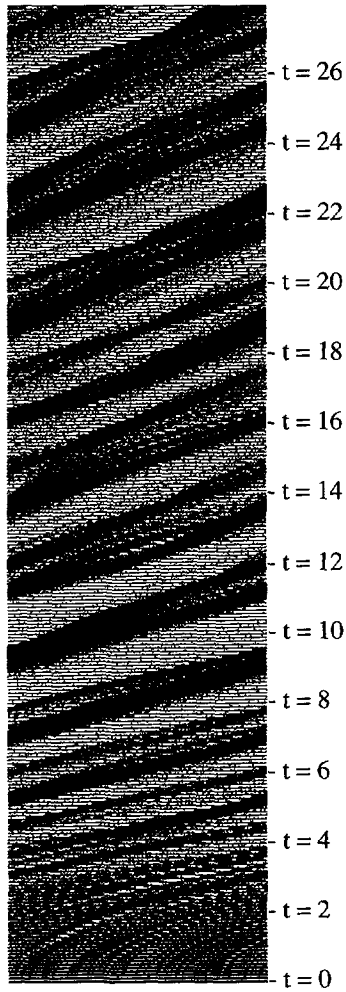

Fig. 5. — Snapshots of the pipe each 1,000 time-steps. The time is given in 10^4 time steps. The density fluctuations are not equidistant and not stable. They can converge as well as diverge. Dark regions correspond to higher densities. The particles of which the wall consists of are not drawn.

of the velocity of a liquid flowing through a pipe [23] while the pressure is increased gently. At low flow velocity an incompressible viscous fluid forms a laminar flow (Poisseuille–flow) which gets faster with rising energy. If the velocity reaches a certain value u_c the Poiseuille–flow becomes unstable and an inhomogeneous flow regime becomes stable. For the case of a fixed pipe geometry and given fluid parameters the critical velocity of the flow u_c is described by the critical Reynolds–number [24] $\mathrm{Re}_c = \frac{u_c \cdot D}{2 \cdot \mu_k} \approx 2,300$ where μ_k is the kinematic viscosity of the fluid. Provided there are critical fluctuations the system turns into a non–laminar regime. If the pressure is increased very gently and the system is not disturbed in another way, however, it is possible to maintain the Poiseuille–flow at velocities far over Re_c in the unstable regime. Then a small fluctuation causes the system to turn to the stable state, i.e. it suddenly lowers the velocity of the flow and dissipates the energy. As shown for the flow of a granular material there exist different flow regimes too and the transition between them is due to a sudden energy dissipation. Hence the transition from low energy to high energy regime of granular flow in a pipe is similar to the transition of a laminar fluid-flow into a non–laminar one.

Experimental and recent numerical investigations of traffic flows [25] lead to similar results as those presented here. At a given average car-velocity the homogeneous traffic flow becomes unstable and traffic jamming occurs provided there are random perturbations of the velocity of the cars which correspond to the collisions of the grains with the wall in our case.

Concluding we state that granular material falling through a narrow pipe appears effects far from the behaviour of a laminar fluid. At a certain energy the granular flow transits from the homogeneous to an inhomogeneous flow-regime where clogging and density waves can be observed. The regions of high particle density can converge as well as diverge, their distances vary irregularly. The results of the experiment shown in the beginning agree with the numerical simulations.

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