Dripping faucet in extreme spatial and temporal resolution

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Besides its importance for science and engineering, the process of drop formation from a homogeneous jet or at a nozzle is of great aesthetic appeal. In this paper, we introduce a low-cost setup for classroom use to produce quasi-high-speed recordings with high temporal and spatial resolution of the formation of drops at a nozzle. The visualization of the process can be used for quantitative analysis of the underlying physical phenomena.
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I. INTRODUCTION

The formation of drops of liquids is an everyday phenomenon, yet obtaining a detailed understanding of the process is not straightforward from an experimental or a theoretical point of view. In engineering, drop formation is of importance for a number of processes such as ink jet printing, spray drying, spray coating, and others.

To observe the process of drop formation at a faucet in detail, the time and length scales have to be resolved appropriately. The duration of the pinch-off process of about 1 ms and the required spatial resolution of about 1.5 megapixels (see Sec. III below for an estimate) determine the requirements for the apparatus. Most modern experiments use high-speed cameras with advanced technical specifications that are far too costly for typical educational budgets. In order to bring to the classroom the phenomenon of drop breakup, which is of high educational value and great aesthetic appeal, we exploit the idea of stroboscopic recording that was developed by Lenard 130 years ago.[1]

The aim of this paper is to describe a low-cost experimental setup capable of recording periodic processes at time resolutions of up to approximately 3×10^5 frames per second (fps) and almost any spatial resolution, limited only by the resolution of an ordinary CCD camera. We will demonstrate that this setup allows high-quality videos to be produced of the pinch-off process of a drop falling from a faucet. The experimental setup is used in undergraduate lab courses on experimental physics, fluid dynamics, nonlinear dynamics, and measurement technology.

The formation of drops from a homogeneous jet, at a dripping faucet, or in splashes has been considered by some of the most eminent scientists in history. It was Mariotte who first described the disintegration of a homogeneous flow of water into drops under the influence of gravity in his seminal book "The Flow of Water and other Fluid Bodies" in 1686.[2] He understood that the flow becomes thinner due to gravity and explained the phenomenon of drop formation by stating that the "elements of the fluid" cannot be too distant from one another. As a qualitative explanation, this is not far from what we today call surface tension, described by Laplace[3] and Young[4] more than a century later. It was Plateau[5,6] who explained the instability of a homogeneous jet by the action of surface tension.

Systematic experimental investigations of drop formation from a jet were first performed by Savart in 1833.[7] Among others, Savart made two important observations: (i) the jet breaks into approximately identical drops, but additionally, depending on the parameters of the experiment, one or more much smaller satellite drops may appear for each drop (see Fig. 1 below); and (ii) small perturbations of the surface of a fluid cylinder grow over time if the wavelength of the perturbation exceeds a certain value. The latter observation allowed for the systematic investigation of the instability of fluid surfaces and gave rise to a large body of related experimental work by Hagen,[8,9] Magnus,[10] Rayleigh,[11,12] and many others.

The formation of satellite drops attracted much attention as it is a non-linear effect and therefore not covered by the linear theory developed by Rayleigh in 1879.[13] Although in static or quasi-static situations the physics of fluid surfaces is governed by an equilibrium of surface tension and gravity, Rayleigh noticed that in rapid processes the relationship between surface tension and inertia dominates.[12,13] Thus, when fluid flows slowly out of a faucet, the characteristic shape of the slowly forming pendent drop[14] is determined by the balance of gravity and surface tension and the dynamics is quasi-static. As the drop grows, the situation becomes unstable and the drop develops a neck that grows in length and shrinks in diameter accordingly. At the moment of breakup, the elongated neck quickly recedes. This highly dynamic process, which is of very short duration, was first experimentally observed in 1887 by Lenard.[1] As a result of this pinch-off phenomenon, we observe complex oscillations that ultimately result in one or more satellite drops (which were investigated in detail by Guthrie in 1863 (Refs. 15 and 16).

There is a large body of literature following this pioneering work, meaning that we now have a good understanding of the process. Eggers and Dupont[17,18] proposed a self-similar description for the time-dependent shape of the neck whose single parameter contains the density, viscosity, and surface tension of the fluid. This result was confirmed by many experimental investigations for both liquid jets (e.g., Ref. 19) and dripping faucets (e.g., Ref. 20). The scenario of satellite droplet formation follows a highly complex hierarchical scheme.[21]

A comprehensive review on the physics of drop formation due to instability of surfaces, including many historical remarks, can be found in Refs. 22–24. For an introductory text discussing many important physical arguments, see Ref. 25.

II. PHOTOGRAPHIC IMAGES OF DRIPPING FAUCETS

The dynamics of drop formation can only be observed by the naked eye to a limited extent.[7] Much of the dynamics and, thus, of the physics of the problem would remain unrevealed

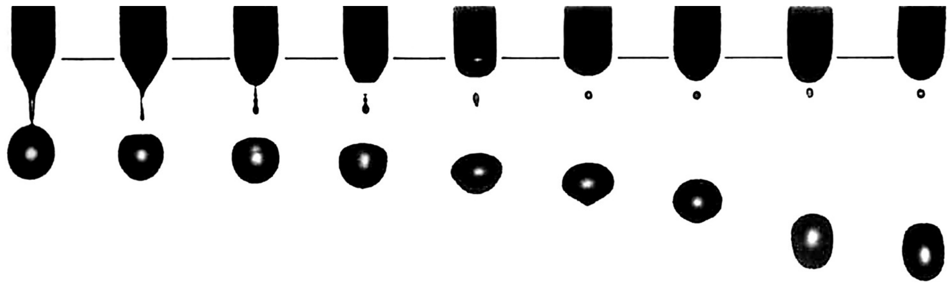

Fig. 1. In 1887, using a stroboscopic technique, Lenard obtained sequences of drops at high time resolution. Shown here is a section of Fig. 3(a) from Ref. 1.

without sophisticated imaging techniques. Given that the collapse of the neck is a very fast process that typically takes a millisecond or less, mechanical shutter cameras are not suitable for recording this process. Therefore, Worthington[26] illuminated splashes of water drops with electrical sparks of very short periods of time (typically $1\,\mu$s) in an otherwise dark room. When impacting the target plate, the drop interrupted an electrical circuit, thus igniting the spark through its own weight. Using this technique, which he called *instantaneous photography*, Worthington obtained splashes of great beauty;[27–29] his paper of 1897 (Ref. 28) contains more than 150 pictures of splashes and he later published an entire book[30] containing mainly photographs of splashes. The beauty of splashes created by the impact of an object on a liquid still fascinates scientists today (e.g., Refs. 31–33).

The first true dynamic recording of a dripping faucet was obtained in 1887 by Lenard,[1] who noted that the time lag between successive drops is highly constant, provided that the inflow of fluid is constant. Therefore, he used a stroboscopic method[1] where a drop was used to trigger the ignition of the flash for the subsequent drop. By adjusting the vertical position of the camera, he was able to record the drop at different stages of evolution after it was released from the faucet. In this way, he recorded a stroboscopic film of the entire sequence at high time resolution (see Fig. 1).

True high-speed films allowing non-periodic processes like splashes to be investigated became available in the 1930s mainly due to the pioneering work of Edgerton,[34] who recorded dripping faucets at up to 1200 fps.[35,36] Edgerton's technique and several applications are explained in an impressive video that is available online.[37] Due to the great progress in the field of imaging technology in recent decades, it is now possible to produce high-speed recordings with high optical resolution and large frame rates. Such recordings can be used to explore the dynamical behavior of a dripping faucet in great detail (e.g., Refs. 20, 38, and 39).

III. SYSTEM REQUIREMENTS

Let us first estimate the technical data of a system necessary to record the drop formation. The fastest process of interest is the collapse of the neck and the corresponding formation of satellite drops, which takes place in about 1 ms. If we wish to stretch this time to 10 s of slow motion at 30 fps, we need a recording frame rate of 300,000 fps. The smallest details we wish to resolve are the pinch-off regions, which have a typical size of 0.2 mm [see Fig. 4 (inset)]. If we wish to resolve these features with 50 pixels, a desired field of view of approximately $8\,\text{mm} \times 12\,\text{mm}$ corresponds to a sensor of 2000×3000 pixels in size. High-speed digital cameras with these characteristics are not currently commercially available. Therefore, the idea of stroboscopic recording, introduced by Lenard,[1] is applied with some modifications. To enable precise timing, the free running interruptor was replaced by a light barrier that is broken just before the pendent drop falls. The light barrier triggers a sequencer that, after a certain delay, ignites the flash and thus exposes the picture. By incrementing the delay between breaking the light barrier and firing the flash, a series of pictures can be recorded that, when played back at a fixed frame rate, reproduce the process of detachment of the drop in slow motion. Special attention has to be paid to the flash bulb; in order to obtain sharp pictures, no object should move by more than one pixel during the time of illumination. We assume a typical neck length of 4 mm (see Fig. 4) corresponding to about 400 pixels and a collapse time of about 1 ms. If we assume constant velocity of the neck during collapse for an estimate, the maximum exposure time is thus $\left(10^{-3}/400\,\text{px}\right) \times 1\,\text{px} = 2.5\,\mu$s.

IV. SPECIFICATION OF THE SYSTEM

Figure 2 shows a CAD drawing of the setup, and Fig. 3 shows the actual apparatus. The fluid is initially contained in

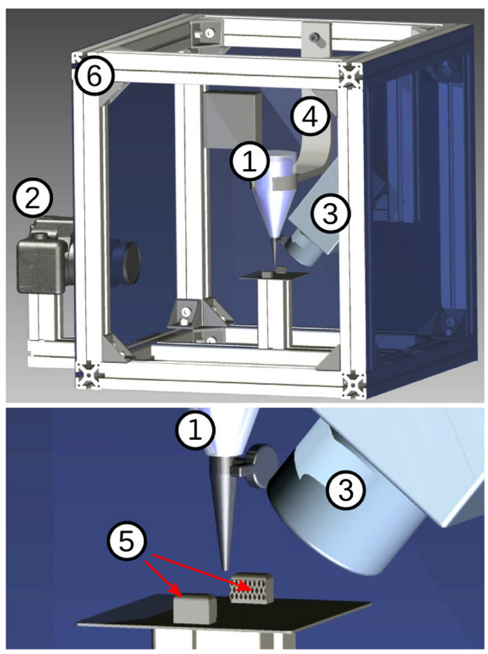

Fig. 2. CAD drawing of the experimental setup: ① glass tank with valve and faucet, ② digital camera, ③ flash bulb, ④ linear actuator, ⑤ light barrier, and ⑥ aluminum frame. The lower collecting tray, control for the flash bulb, function generator, and power supply are not shown. The bottom panel is a close-up of the main figure.

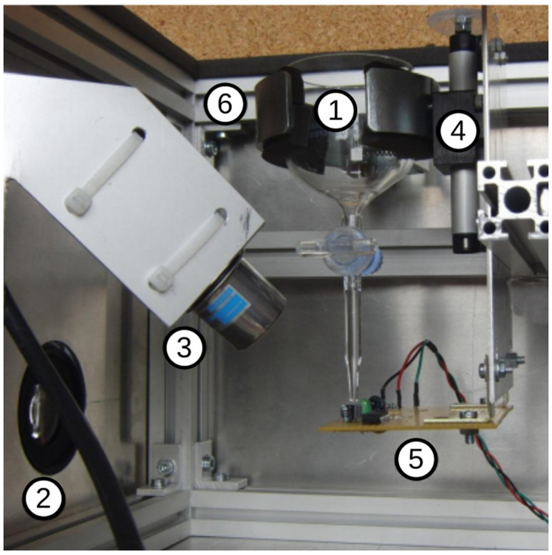

Fig. 3. Photograph of the actual apparatus. The labels are the same as in Fig. 2: ① glass tank with valve and faucet, ② digital camera, ③ flash bulb, ④ linear actuator, ⑤ light barrier, and ⑥ aluminum frame.

a glass tank ① connected to the faucet via a valve to adjust the drop rate to about one drop in 10 s. It is favorable to set the drop rate to a low rate, because this allows surface ripples on the fluid portion remaining at the faucet to die out so that they do not cause perturbations of the forming drop by internal currents. A linear actuator ④ allows precise control over the distance between the faucet and the light barrier ⑤ such that the forming drop, which is a pendent at the faucet, interrupts the light barrier a few milliseconds before breaking off. The signal from the light barrier (e.g., U-shaped photoelectric sensor Panasonic PM-25) starts a triggerable sequencer (TimeMachine[40] or any triggerable function generator). The sequencer then produces a pulse following the trigger signal with a certain delay to ignite the flash bulb ③, exposing the picture. In the time prior to the flash, the camera shutter ② is held open, and only after the flash has fired is the shutter closed and the image recording complete. To this end, the camera is operated in the bulb mode and the release of the shutter is also controlled by the sequencer. The entire setup is mounted in a case of aluminum elements with opaque sides to have the flash bulb as the only source of light.

The digital single-lens reflex (DSLR) camera used in the setup has a resolution of about 3000×2000 pixels spaced on an APS-C sized sensor. In combination with a macro lens (reproduction ratio 1:1), this yields a resolution of about $8\,\mu$m/pixel in the imaged plane of the droplet. We used a short-arc xenon flash bulb (L11946, socket E10977 and driver C10980 from Hamamatsu) to produce flashes with a duration of $\Delta t \approx 2\,\mu$s, fulfilling the condition above and delivering exceptionally sharp images. The brightness of the picture is determined by the quantity of light, i.e., by the total energy released during a single flash. The driver connected to the flash bulb was configured to provide $E = 0.5$ J per flash, enough to obtain a well lit image given the short distances of the setup.

V. EXAMPLE

Figure 4 shows a photograph of a water drop with a diameter of approximately 4 mm at the instant when it detaches

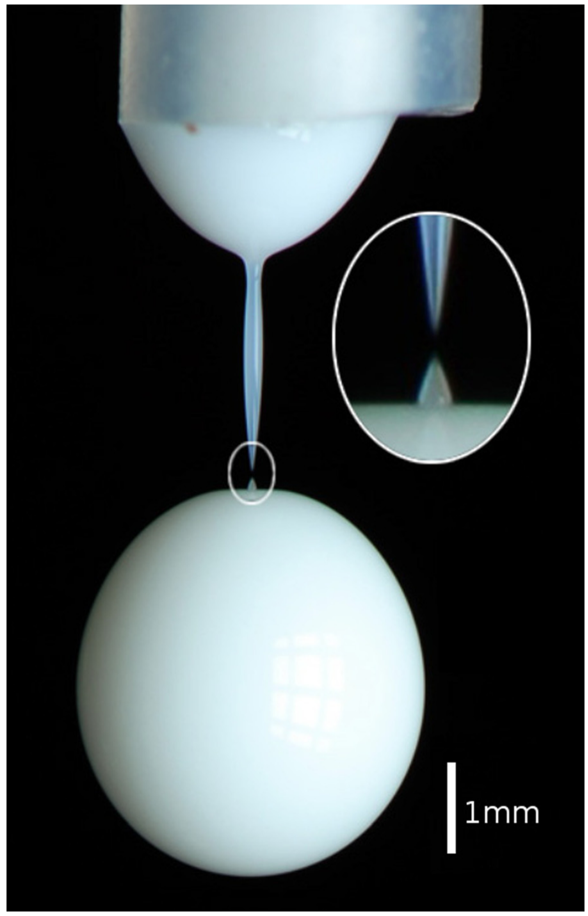

Fig. 4. Photograph of a water drop with a diameter of approximately 4 mm at the instant when it detaches from the faucet. For better contrast, a small amount of milk has been added. A coarse grid was placed in front of the flash bulb to visualize the curvature of the droplet at the reflection point of the light source. The inset shows a close-up view of the pinch-off region.

from the faucet. The inset, showing a magnification of the pinch-off region, demonstrates the high resolution. The fluid is water with about 5% of milk added for better contrast. (We noticed later that in the original literature both Worthington and Edgerton[26,37] likewise used milk for contrast.) Figure 5 shows a sequence of freeze frames from a sequence recorded by the described stroboscopic method, together with close-ups of the neck area showing the formation of a satellite drop and its subsequent highly complex oscillations. The full video sequence can be seen as an online enhancement to Fig. 5. (The video was reconstructed from a sequence of 463 frames of consecutively detaching drops.)

We wish to draw attention to an interesting feature seen in Fig. 5 and impressively demonstrated in the movie. After the elongated neck is pinched off at both ends, it collapses into the satellite droplet, which then moves *upwards* against the direction of gravity and eventually reaches the faucet (not shown in Fig. 5 but can be seen in the movie). This observation is quite reproducible but may appear counter-intuitive at first glance; after all, starting from the time of the pinch-off the only external force acting on the satellite is gravity. The reason for this motion is that the elongated neck is pinched off from the falling drop slightly earlier than from the faucet (see Refs. 18 and 20 for a mathematical analysis). As soon as the neck is pinched off from the drop, its lower part accelerates upwards driven by surface tension. But why does it not

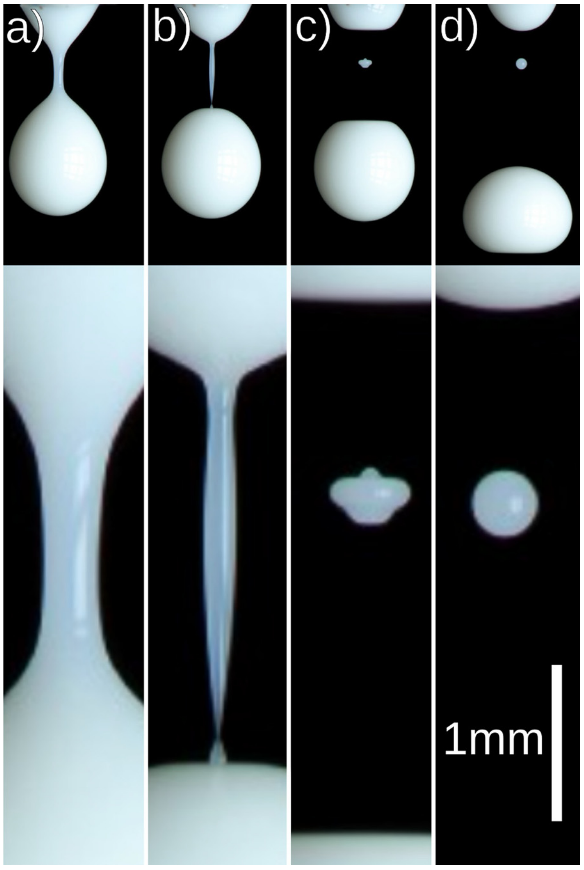

Fig. 5. Freeze frames of a sequence recorded at (a) $t = 0$ (arbitrary starting time), (b) $t = 1.70$ ms, (c) $t = 2.98$ ms, and (d) $t = 12.38$ ms. Panel (b) corresponds to Fig. 4. The full video sequence can be found as an online enhancement (enhance online)[URL: http://dx.doi.org/10.1119/1.4979657.2].

snap back to the faucet? To understand this, we must consider the inertia of the neck. Snapping back means that the material forming the neck is accelerated upwards. The force due to surface tension is large but finite, so it would take some time for the liquid in the neck to arrive back at the faucet. The mathematical analysis shows that the pinch-off process at the upper end is faster than the time required for the neck to recede. When pinched off from the faucet, the upper end of the neck accelerates downwards (towards the center of mass of the neck), also driven by surface tension. At the moment when the neck is pinched off, its total momentum is directed upwards such that the resulting satellite drop moves upwards due to its momentum. Of course, at some point gravity would reverse the direction; however, before gravity can compensate for this motion, the satellite reaches the faucet. Thus, we see that the upwards motion of the satellite drop is a result of the interplay of gravity, surface tension, and inertia.

For typical recordings, the time delay between breaking the light barrier and triggering the flash by the sequencer was increased by ~50 μs from one frame to the next, corresponding to a frame rate of 2×10^4 fps. Note that the frame rate can be increased to almost any value, limited only by the resolution of the sequencer, the duration of the flash, and, of course, fluctuations in the drop formation (for instance, due to mechanical perturbations). The release of the drop causes small vibrations of the nozzle and the fluid surface of the next drop pendent on the nozzle, which is why we adjusted the flow rate such that a drop detaches approximately every 10 s. Therefore, the recording time for a process of real time $t_{\rm real}$ is $t_{\rm rec} = (10\,{\rm s}/50\,\mu{\rm s}) \times t_{\rm real} = 2 \times 10^5 \times t_{\rm real}$. For instance, the recording of the video corresponding to the sequence shown in Fig. 5 with a real time duration of $t_{\rm real} \approx 15$ ms took about $15 \times 10^{-3}\,{\rm s} \times 2 \times 10^5 = 50$ min. The recording could be accelerated by improving the mechanical support, which would be desirable in order to keep the external conditions such as room temperature and air pressure constant during the recording. Note, however, that the time delay between consecutive drops falling from a faucet is only nearly constant at low flow rate, which implies simple periodicity of the process (one drop per period). For higher flow rates, the process reveals a higher-order periodicity and eventually becomes chaotic.[41,42] For the method employed here, it is absolutely essential that the drops are almost perfectly identical, and this would not be the case for a flow rate that were too high.

VI. QUANTITATIVE RESULTS

In this section, we demonstrate how to obtain quantitative results from the recorded images shown above.

A. Surface tension

As a simple example, we show how to measure surface tension. The force that prevents the drop from falling when still hanging at the faucet is due to surface tension. When water flows from the faucet, the drop grows and thus gains mass until the weight exceeds this force and the drop pinches off. The quantity that quantifies the ratio of weight and the force due to surface tension is the *Bond number*[43]

$$\mathrm{Bo} \equiv \frac{\rho g l_k^2}{\sigma}, \tag{1}$$

where ρ is the density of the fluid, g is the gravity, l_k is the characteristic length of the fluid body (the capillary length), and σ is the surface tension. For $\mathrm{Bo} \ll 1$, surface tension dominates and for $\mathrm{Bo} \gg 1$, gravity dominates. The condition $\mathrm{Bo} = 1$ is, thus, a criterion for the balance of surface tension and gravity and delivers the capillary length

$$l_k = \sqrt{\frac{\sigma}{\rho g}}, \tag{2}$$

which is of the order $l_k \approx 2.7$ mm for the values of surface tension and density for water. Hence, the drop pinches off not much earlier than characterized by $\mathrm{Bo} = 1$ since surface tension keeps it at the faucet, but also not much later, since gravity then takes over. Thus, the typical drop diameter is expected to be $D = l_k$. Measuring the size of the drop falling from a faucet therefore allows the surface tension of the fluid to be determined. For the experiment, we prefer another representation

$$\sigma = \frac{6^{2/3} \rho g}{\pi^{2/3}} V^{2/3}, \tag{3}$$

which is obtained from Eq. (2) when the diameter $D = l_k$ of the drop is replaced by its volume $V = \pi D^3/6$. The reason for preferring Eq. (3) is that after pinch-off the drop performs prolate/oblate oscillations such that the diameter is not well defined. In contrast, the volume can always be determined from the images due to the central symmetry of the problem.

We determined the volume for typically 200–400 images taken in one measurement to obtain the mean volume $\bar{V}$ of a drop. Here, we present results for water and a mixture of 80% water and 20% glycerol, indicated by subscripts w and g, respectively. In both instances, we added a very small fraction of milk for good optical contrast. For water, we obtain $\rho_w = 999.6\,\text{g/cm}^3$, $\bar{V}_w = 10.61\,\mu\text{l}$, and with the help of Eq. (3), $\sigma_w = (7.29 \pm 0.01) \times 10^{-2}\,\text{N/m}$, which is remarkably close to the value for water at 20 °C in contact with air, $\sigma = 7.28 \times 10^{-2}$ N/m (see Ref. 44). The uncertainty of the measurement given here is the standard deviation obtained from a set of 100 independent measurements (different drops). For the glycerol-water mixture, we find $\rho_g = 1.046\,\text{g/cm}^3$, $\bar{V}_g = 9.5\,\mu\text{l}$, and finally $\sigma_g = (7.09 \pm 0.01) \times 10^{-2}\,\text{N/m}$, which agrees also with the value from the literature, $\sigma_g = 7.112 \times 10^{-2}\,\text{N/m}$ (see Table 35 in Ref. 45).

Another approach to measuring surface tension exploits an expression for the period τ of the oscillation of the falling drop of volume V provided by Rayleigh[11]

$$\tau = \sqrt{\frac{3\pi}{8}\frac{\rho V}{\sigma}}. \tag{4}$$

The period τ of the prolate-oblate oscillation can be obtained from video sequences such as the one provided as an enhancement to Fig. 5.

B. Neck thinning near breakup

The pinch-off process reveals remarkable scaling relations, based on self-similar properties that have been found using numerical simulations, theoretical modeling, and experiments.[18,20,22,39] An important example is the dependence of the smallest width of the neck on the time Δt remaining until pinch-off, which obeys a power law

$$h_{\min}(\Delta t) \sim \Delta t^{2/3}. \tag{5}$$

Figure 6 shows the results of our experiments for the same fluids as in Sec. VI A. Again, we find good correspondence with the results from the literature (see, e.g., Fig. 4 in Ref. 39 and Fig. 31 in Ref. 22, where a full discussion of the scaling properties can also be found).

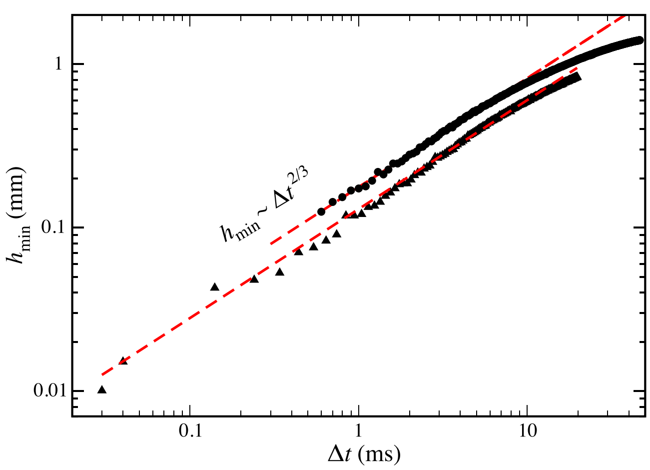

Fig. 6. Minimum neck diameter $h_{\min}$ as a function of Δt, the time remaining until pinch-off. The circles show the results for water; the triangles for a mixture of 20% glycerol and water. The dashed lines (red online) lead the eye to the power law $h_{\min} \sim \Delta t^{2/3}$.

VII. CONCLUSION

We introduced a setup that can be used to produce stroboscopic high-speed recordings of the formation of drops at a nozzle with high temporal and spatial resolution that would not be achievable otherwise. We analyzed the performance of the system and its limitations and provided an example application. In principle, the system (possibly with mechanical modifications) can be applied to produce high-speed recordings of *any* periodic process.

The total cost of the components of the setup is about $900, including the short-arc xenon flash bulb and driver (~$650), a triggerable function generator, case material, glass pipette, and small mechanical components, but excluding the camera, which is a standard 6 megapixel model. For our application, this should be compared to the costs of a high-speed camera capable of recording videos at a resolution of 300,000 fps and 2000 × 3000 pixels.

Based on sequences of photographs of the type shown in Figs. 4 and 5 and the corresponding video sequences, we can now apply methods of image processing to obtain quantitative data. In this paper, we provide two examples, both of high educational value: (i) we deduce the surface tension from the measurement of the volume of the particles and (ii) we discuss a prominent feature characterizing the scaling relations of the problem; that is, the nature of its universality.[17] Both measurements have been performed with two different fluids, and in all cases, we obtained very good correspondence with results from the literature.

But in addition to the physics, one can also simply enjoy the great aesthetic appeal of a simple dripping faucet that only becomes apparent when watching it at high time resolution.

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