Spontaneous Size Segregation in Textured Granular Pipe Flow

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Abstract

A new spontaneous particle segregation mechanism is found in textured granular pipe flow. In contrast to the previous studies in similar geometry, where the particle segregation is caused by air drag differences of different grain sizes in the pipe flow, in this study we found, even in an air-less environment, the particle segregation can be triggerred by the perturbations imposed from the particle collisions on the textured confinement wall. The pipe inner walls in our simulations are assumed frictional, as well as for the embedded wall textures. This system conformation provides external perturbation input to the falling granular particles in the pipe. Previous studies of textured pipe flow have found, for a monodisperse system, the spiral inner wall texture smoothes density waves and jamming plugs. We discover that, for a binary mixture system, the wall spiral textures do not only smoothen the bulk density waves, but also provide effective shaking to the system. This spontaneous shaking causes perturbation input in different frequencies, depending on the inner wall structures. Because particles of different sizes respond differently to this shaking, this effect eventually leads to spontaneous size segregation.

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I. INTRODUCTION

Granular media is a term that denotes a general class of substances of great importance, both in industry and everyday life. They are extensively refer to collections of distinguishable solid particles. The spectrum of industrial applications relating to granular materials indeed ranges from the food granules processing in eddy agglomerators [1], to the admixture of very fine rubber powders in the manufacture of car tires. However, due to the complex nature of granular particles, i.e. its bulk material properties can be either considered as solid, liquid or gas depending on its energy and packing state, there is no unified theory for describing its transportation process. Granular dynamics is certainly not only of scientifically interests, since expenses of billion dollars are spent every year in industries to solve the granular transportaion related problems.

In simplest geometries, such as granular materials of monodisperse or polydisperse distributions flowing down pipes, their transport properties are already intrinsically unstable and complex. Among many resulting dynamic phenomena, the travelling density waves could cause either clogging, which slows down the material flux, or in worse case it generates jamming that blocks the granular flow completely.

It is always challenging to precisely predict the mass flux of the granular materials that flowing throught a pipe, i.e. the mass flux does not have a simple dependence on the pipe diameter and material volume fraction, but it is rather complicately depending on the combination of particle shape, size dispersity, material properties and aspect ration of pipe and grains [2]. To regulate the granular pipe flow, it was shown that an obstcle insertion above the silo outlet significantly reduces the blocking probability [3]. On the other hand, through an application of electric fields or an inclusion of external mechanical perturbations, the jamming problem can also be resolved. In fact most of the proposed strategies of smoothing granular pipe flow involves energy input from external sources. Previously, it was demonstrated the mass flux can be homogenized and the flow blockage is avoided when the pipe inner-wall is embedded with helical texture. This study found that, in a monodisperse granular system, the spiral-type wall texture smoothes the density waves hence greatly resolves the granular plugs [4].

Most of the natural granular materials, however, are not monodisperse, i.e. the particles follow a certain size distribution. It is hence of essential importance to study the transport dynamics of polydisperse granular systems, in order to account for the real-world situations. In this investigation, we focus on the dynamics of gravity-driven polydisperse granular pipe flow. Different inner-wall textures of the tube will also be used, in order to find out the wall effects on influencing the bulk dynamics. The goals of this study hence are: 1) to find out the mathematical description of density wave formation and their propagation dynamics, and 2) how does the particle dynamics modified by the the imposed inner-wall textures.

In the following we will first describe our system and the simulation model, the discrete element method (DEM). This model is essentially a molecular dynamics algorithm in which the interactive particle potentials are replaced by surface contact model. Secondly, the similarity of granular pipe flow and real world traffic system will be compared. The mathematical expression of the density wave propagations will be discussed. We will show the classification of forward/backward propagating waves can be determined, in the phase space of pipe solid fraction and aspect ratio of paricle size and pipe diameter. The binary granular system is then studied in Sec.V. We found the wall textures impose effective shaking to the granular pipe flow systems. This binary granular system undergoes size segregation under certain conditions, and its mechanism is then discussed.

II. NUMERICAL MODEL

The numerical algorithm used here is Discrete Element Method (DEM), in which method the Newtons equations of translational and rotational motion are solved simultaneously for all particles. The collision model is the core that determines the particle contact physics, we use the viscoelastic Hertz contact model in the normal direction, which excludes the cohesive force during contacts. For the contact force in tengential direction, a modified CundallStrack model is applied. The normal contact force between two elastic spheres collision is:

$$\vec{F}_n = min(0, -k\xi^{3/2} - \frac{3}{2}A_n k\sqrt{\xi}\dot{\xi})\vec{e}_n \tag{1}$$

and the compression of particles at positions $\vec{r}_1$ and $\vec{r}_2$

$$\xi = R_1 + R_2 - |\vec{r}_1 - \vec{r}_2| \tag{2}$$

The elastic constant k in Eq.(1) is a function of the Poisson's ratio ν, the Young's modulus Y, and the effective radius $R_{eff} \equiv R_1R_2/(R_1R_2)$.

$$k \equiv \frac{2Y}{3(1-\nu^2)}\sqrt{R_{eff}} \tag{3}$$

It is noted that the kinetic energy can be dissipated during binary collision, for which it is controlled by the dissipative parameter A_n. This dissipative parameter A_n has a complex dependence on the material viscosities and is usually not directly available. On the other hand, the elastic constant k can be directly computed from material properties and easily available in material table.

In non-headon collisions the tangential contact force plays important role. The modified Cundall-Strack model has the following expression:

$$\vec{F}_t = min\left[\mu|\vec{F}_n|, \int_{path} \frac{4G}{2-\nu}\sqrt{R_{eff}\xi}ds + A_t\sqrt{R_{eff}\xi}v_t\right]\vec{e}_t \tag{4}$$

in which μ is the Coulomb friction coefficient and the shear modulus G is given by the equation $2G = Y/(1+\nu)$. In Eq.(4) the displacement of the particles at the point of contact is integrated for the duration of the contact. Relative tangential velocity $\vec{v}_t = v_t\vec{e}_t$ is the transverse velocity perpendicular to the vector connect the centers of sphere's mass. The surface roughness determines the tangential dissipative parameter A_t, and this value is chosen to satisty a condition, for which the normal and tangential deformation rates are of the same order of magnitude. With this assumption it was found the simulation results have excellent agreements with expeimental values of particle velocity [5]. We use $A_t \approx A_nY/(1-\nu^2)$ in our numerical model.

The simulation tool used in this investigation is the open source software LIGGGHTS

Parameter	Symbol	Value
Particle material density	ρ_p	2650 $kg\ m^{-3}$
Particle diameter	d	2.4 mm
Young's Modulus	Y	10^8 Pa
Poisson's ratio	ν	0.24
Coulomb's friction coefficient	μ	0.5
Coefficient of restitution	e	0.5
Pipe length	L	1 m
Timestep	Δ t	5×10^{-7} s

TABLE I: Numerical values of the parameters used in simulations [4]

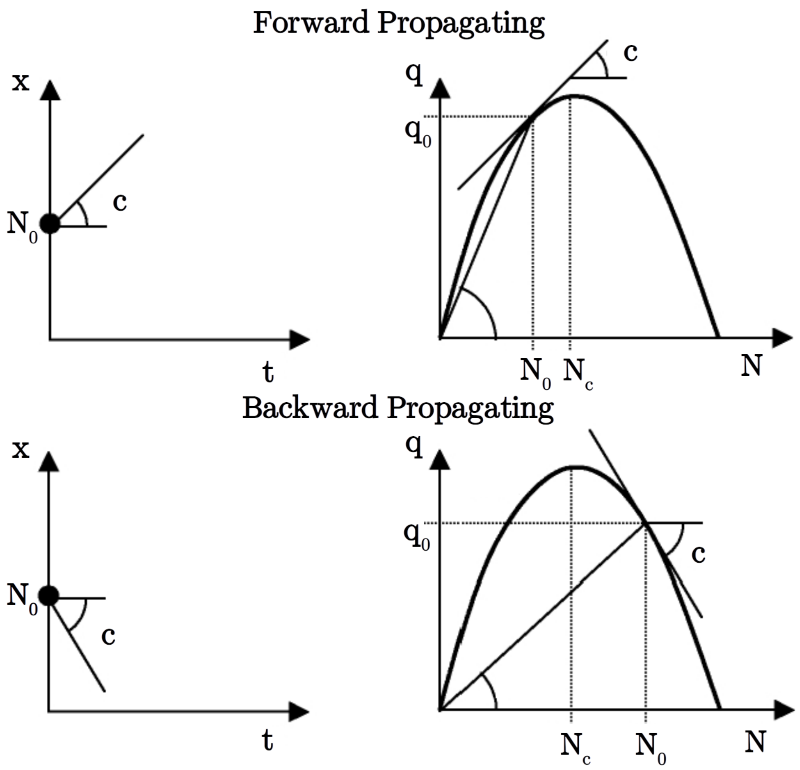

FIG. 1: The prapagation of density waves in granular pipe flow, assuming conservation of mass and non-distortion density profile.

[6]. For simplcity the wall material is assumed to be the same as the granular particles, therefore the equations used for computing the wall-particle interactions are the same as those for particle-particle interactions. The material and simulation parameters are listed in Table I. The details of selection of simulation parameters can be found in the study of using helical inner-wall texture to prevent pipe flow jamming [4].

For the simulation geometry, the pipe has a circular cross-section with diameter D mm and the pipe length is set to $L = 1$ m. Initially our particles has random radial velocity between $-v_t$ and v_t, where $v_t = 0.014$ m s^{-1}, and they are uniform randomly located along the pipe. It is noted that we have confirmed the finite size effect is excluded in this simulation setup, and this was confirmed by our test simulation runs using longer pipes.

III. FUNDEMENTAL DIAGRAM OF MONODISPERSE SYSTEM

We first look at the frequently observed density waves in granular pipe flow. Due to the similar nature that objects are moving in confined linear geometery, real world traffic flows have been studied using simplified hydrodynamic model [7]. In that work the author

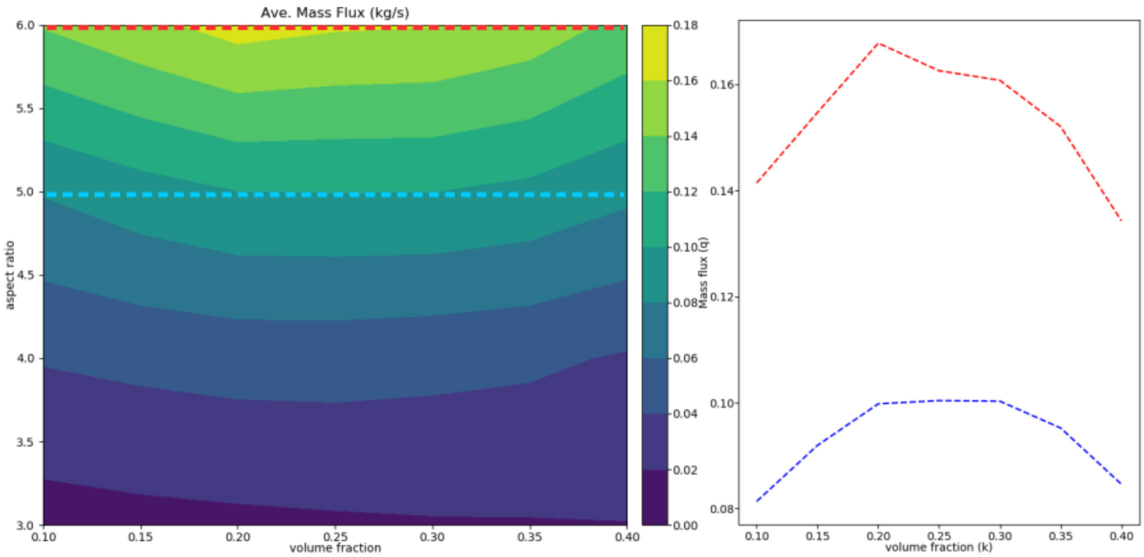

FIG. 2: Mass flux of monodisperse granular flow in non-texture pipe, plotted in the space of volume fraction and particle/pipe aspect ratio. The 1D profile plot for aspect ratio $D_\phi = 5$ (blue dashed line) and $D_\phi = 6$ (red dashed line) are drawn in the right panel.

postulated a linear relationship between flow speed and particle number density.

$$\vec{q} = N\vec{V} \tag{5}$$

where $\vec{q}$, N and $\vec{V}$ are the density flux, particle number density and bulk velocity of granular flow.

The studied system, although it looks seemingly simple, is actually complex to be considered using hydrodynamic conservation laws. This is due to the fact that certain bulk kinetic quantities, such as the internal energy dissipation generated by particle collisions, and the momentum transfer between particles and wall, are in fact comlex physical quantities and have no simple expressions, in terms of other hydrodynamic variables, such as flow velocities or density. This complexity is especially true if the system is in a very dynamic state. Nevertheless, the collective motion of those particles should follow the hydrodynamic continuity equation:

$$\frac{\partial N}{\partial t} + \nabla \cdot (N\vec{V}) = 0 \tag{6}$$

In one-dimension assumption this equation can be further expanded as follows.

$$\frac{\partial N}{\partial t} + \frac{\partial q}{\partial N}\frac{\partial N}{\partial x} = 0 \tag{7}$$

we assume mass flux is a linear function of density, i.e. $\partial q/\partial N = c$, the general solution of this 1-D continuity equation is:

$$N(x,t) = F(x - ct) \tag{8}$$

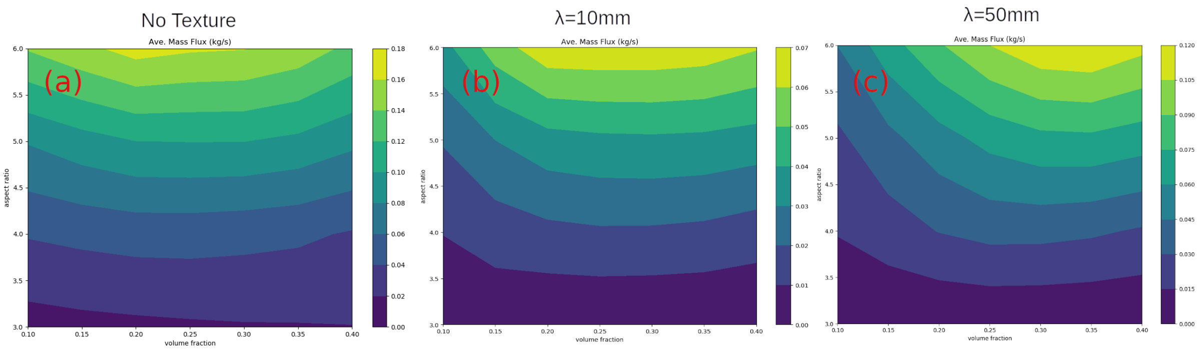

FIG. 3: Mass flux comparison of monodisperse granular systems in no-texture (panel a) and texture pipes (panel b and c).

where $F(x - ct)$ is any function with variable $x - ct$. This function does not distort but rather keep its shape and travel in space as time advances.

It is seen in Fig.1, if c is a positive constant, i.e. $\partial q/\partial N > 0$, the phase velocity of N propagates in the positive x direction hence it represents a forward propagating wave. On the other hand, if c is a negative constant, i.e. $\partial q/\partial N < 0$, it represents a backward propagating wave.

The dynamics of density wave propagation is confirmed in our monodisperse simulations. In Fig.2 the monodisperse simulations are performed, and the time-averaged mass flux are plotted in the phase diagram of particle volume fraction and aspect ratio. To compare with the wave propagation graphs in Fig.1, one can look at the results with fixed aspect ratios. For $D_\phi = 5$ (blue dashed line) and $D_\phi = 6$ (red dashed line) they are drawn in the right panel of Fig.2. The density waves propagate along the flow direction in the positive slope regions, i.e. for volume fraction ($k = 0.1 \sim 0.2$), and the density waves propagate against the bulk flow in the negative slope regions ($k = 0.3 \sim 0.4$).

IV. HELICAL WALL TEXTURES SMOOTHEN THE PIPE FLOW

Previous studies found the helical wall texture resolves the pipe flow plugs. However, the mass flux reduces accordingly because of the equivilant friction imposed from the collision between granular particles and the wall textures. The comparison of time-averaged flux (in unit kg/sec) is shown in Fig. 3 for pipes without (panel a) / with (panel b and c) helical textures. It is seen that: 1) the mass flux decreases with the decreasing helical pitch, i.e. the higher wall friction causes the reduction of the mass flux. 2) the maximum mass flux, in the aspect ratio / solid fraction space, is shifted for different helix pitches.

It is noted here, the pipe flow blockage criteria is $D_\phi = 3$ for pipe without helical texture. This value changes for pipes with helical textures, since the effective pipe width is reduced

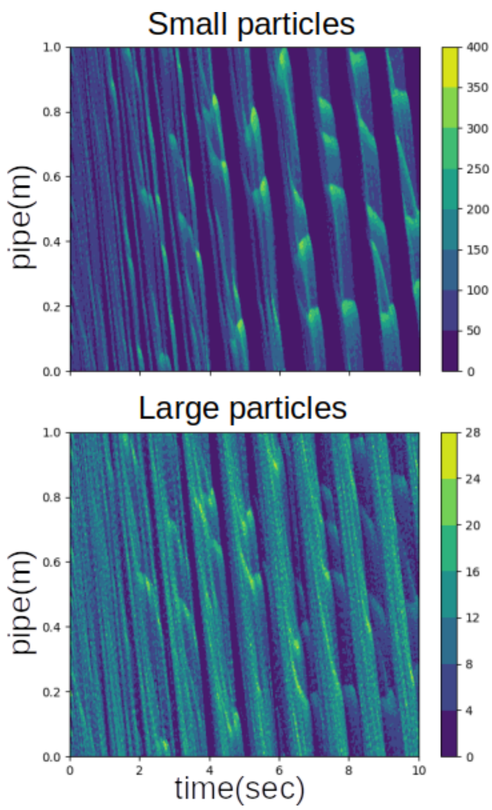

FIG. 4: A typical size segregation of polydisperse granular particles in the pipe flow. The case shown here is for volume fraction $V_\phi = 0.2$ and aspect ratio $D_\phi = 5$. The upper panel is the time evolution of small particles, while the lower panel is for large particles. The colormap indicates the line number density ($\#/m$) of each species.

when the embedded texture particles. This explains the zero-flux region in panel b and c.

V. SIZE SEGREGATION AND ITS MECHANISM

It is very general that granular material is a mixture of different size particles. Its transport dynamics in pipe flow can be influenced by many factors, such as the drag forces from the ambient air or the geometry of the pipe. Both mixing and demixing have their important applications, therefore the control of this process is one of the central issues in granular transport.

In Fig.4 the size segregation is shown for the volume fraction $V_\phi = 0.2$ and aspect ratio $D_\phi = 5$. The upper panel is the temporal evolution for the small particles, and the lower panel is for the large particles. Initially these two species are well-mixed and traveling down the pipe together. After certain time of evolution, vibration on the system is imposed from the particle interaction with the wall, hence the segregation kicked in.

At the first glance the spatio-temporal plots in Fig.4, they look very similar to normal forward propagating density waves. However the detailed comparison of the seperated density, one immediately notice that the regions concentrated with small particles have very few large particles. Vice versa it is also true, that the regions with higher density of large particles

Particle segregation is not caused only by the helix inner wall structure. Our simulation results show that, ring innerwall structure as well as random innerwall structure causes particle segregation in a binary granular mixture. Our speculation is, the particles of different sizes interact differently with the wall texture, and this eventually leads to distict collective dynamics of these two species. Over long time of evolution the two granular species are seperated from each others in the flow.

Previous quasi-2D experimental study has shown that, mixing and demixing of a shaken polydisperse granular system can take place at the same time, and the final state depends on the external driving frequency [8]. In general if the shaking frequency or shaking amplitude increase, the final system state can evolve from segregation crystal, to segregation liquid and eventually as binary gas.

In our granular pipe flow system, the dropping particles interact with wall via collisions. In the presence of helical texture, the texture is made of particles that extruding into the pipe, therefore the granular particles collide with the texture particles while falling.

Due to the difficulty of calculating the wall / granular-particle collision frequency, we treat it as a frequency which is much higher than the texture-particle / granular-particle collision frequency. We assume the number of 2π spiral waves in our tube represents the collision frequency, i.e. higher number of spirals indicate a higher collision frequency for the granular mixture, providing the bulk falling velocity is a constant. On the other hand, if the bulk falling velocity is not a constant, then the effect collision frequency should be defined as number of spiral waves times the bulk falling velicities.

If this assumption is correct, one can calculate the effective collision frequency for helical textured pipe then check if the particle segregation takes place accordingly. In Fig.5 the time-averaged flux is plotted for pipe without texture (panel a) and with helical texture (panel b for wavelength $\lambda = 10mm$ and c for $\lambda = 50mm$). Basically the simulation results resemble to those of monodisperse granular pipe flow. The inner wall texture provides effective friction to the particles, hence the flux decreases with increasing number of spiral waves. The pipe without helical texture allows of course the highest flux. Not shown here, the helical texture has similar effect of resolving the density plugs like for monodisperse system, although it is also observed the flux in polydisperse system is generally larger than the flux in monodisperse system.

Comparison of the volume fraction difference (VFD) of small and large granular particles,

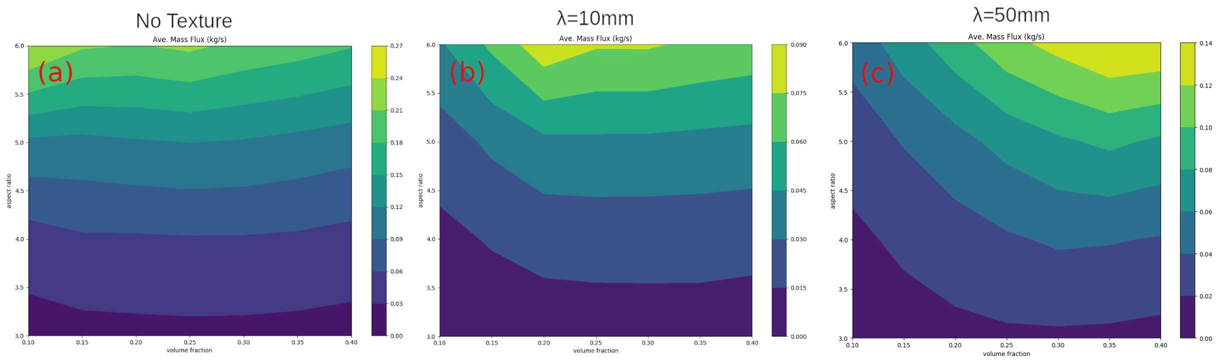

FIG. 5: Mass flux of polydisperse granular flow in pipes without (panel a) and with (panel b and c) helical texture, plotted in the space of volume fraction and particle/pipe aspect ratio.

for the cases with helical wave length with $\lambda = 50mm$ and $\lambda = 10mm$, it is seen that with larger wave length ($\lambda = 50mm$) the system shows more pronounced size segregation, while for the smaller wave length ($\lambda = 10mm$) case the granular mixtures are still mostly preserved as in its original state.

Considering the flux in pipe with $\lambda = 50mm$ is larger than the flux with $\lambda = 10mm$, i.e. $U_{\lambda=50mm} = 1.2 \sim 1.5 \times U_{\lambda=10mm}$, and in the meantime the number ratio of spiral texture is $N_{\lambda=50mm} = 0.2\ N_{\lambda=10mm}$, this leads to the effective collision frequency for the bulk granular system is $f_{\lambda=50mm} = 0.24 \sim 0.3\ f_{\lambda=10mm}$. The significant difference of effective frequency gives the possibility that these two systems locate in different states.

The volume fraction difference maps (VFD) are plotted in Fig.6. It is seen that, if the pipe has no embbeded texture, there is no obvious size seperation as shown in panel a. On the other hand, the size segregation start to take place when the embedded texture impose collision to the granular systems.

VI. DISCUSSION AND CONCLUSION

A novel spontaneous particle segregation mechanism is found in granular pipe flow. In contrast to the conventional particle segregation in similar gravity-driven granular pipe flow, for which the particle separation mechanism is driven by the drag difference of different particle sizes, the new mechanism found here works also in a vaccum environment. Due to the distinct interactions between different particles with the same textured wall, the dynamics of different particles are gradually separated in the transportation process.

It is shown in our simulations, even with collisions between small/large particles, the segregation does not take place in a texture-less pipe flow. However, when the pipe inner-wall is made of spiral textures, the local clusters of same size particles start to appear. More

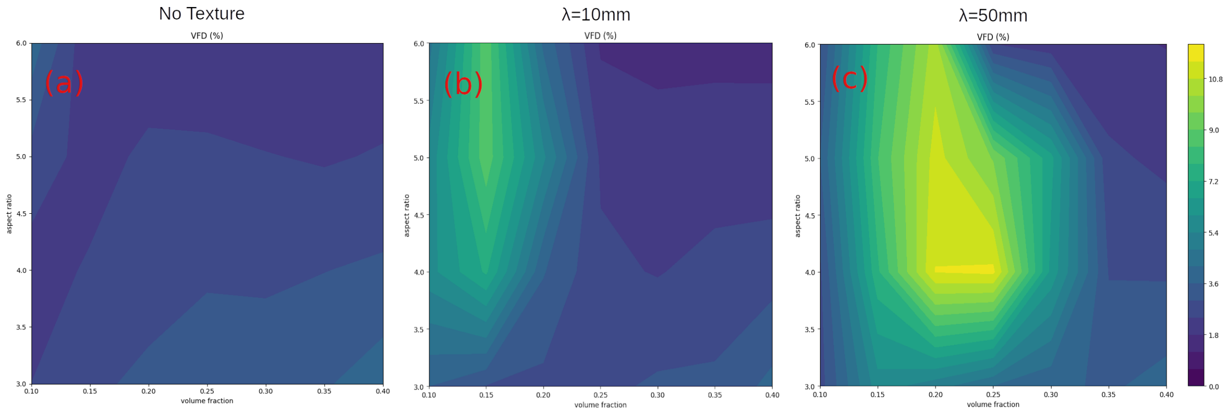

FIG. 6: Volume fraction difference of polydisperse granular flow in pipes without (panel a) and with (panel b and c) helical texture. Lower value indicates the system is in the mixing state, while larger value indicates the system has size segregation.

interestingly, the space occupied by small particles is almost empty of large particles, and vice versa.

On the other hand, the smoothening effect of density waves generated by the helical inner-wall structure, which was originally found in the monodisperse granular particles, still holds true in the current polydisperse system. Therefore we can conclude the swirling motion in general prevents the local density enhancement.

For the dynamics of density wave propagation, we have compared our pipe flow system with vehicle traffic system. These two systems share suppising similarities hence their particle kinematics are interchangeable. For this simplified 1D pipe flow, the hydrodynamic continuity equation should be satisfied. In the phase diagram of filling fraction and flux, it was found the positive slope corresponds to forward propagating density waves, and the negative slope corresponds to backward propagating waves. This analysis can be applied to both traffic and granular pipe flow systems. Because the density wave is granular pipe flow is an important issue in many industrial applications, our study provides physical insights and useful estimations to prevent the potential damage when operating granular pipe flow systems.

[1] Hsiu-Po Kuo, Wei-Ting Tseng, and An-Ni Huang, *Powder and particle*, 33, 239 (2016)

[2] Yinchang Li, Rui Liu, Mahendra Shinde, and Meiying Hou, *Granular Matter*, 14, 137 (2012)

[3] Celia Lozano, Alvaro Janda, Angel Garcimartin, Diego Maza, and Iker Zuriguel, *Physical Review E*, 86, 031306 (2012)

[4] F. Verbücheln, E. Parteli, and T. Pöschel, *Soft Matter*, 11, 4295 (2015)

[5] C. H. Rycroft, A. V. Orpe and A. Kudrolli, *Phys. Rev. E: Stat., Nonlinear, Soft Matter Phys.*, 80, 031035 (2009)

[6] C. Kloss, C. Goniva, A. Hager, S. Amberger and S. Pirker, *Prog. Comput. Fluid Dyn.*, 12, 140 (2012)

[7] B.D. Greenshields, *Proc., 13 th Annual Meeting of the Highway Research Board*, Dec. (1933)

[8] P. M. Reis, T. Sykes, and T. Mullin, *Phys. Rev. E*, 74, 051306 (2006)