Heat conduction due to laser beam heating: effect of surface geometry

Michael Blank  
Institute for Multiscale Simulation  
Friedrich-Alexander-Universität Erlangen-Nürnberg  
Erlangen, Germany  
michael.u.blank@fau.de

Prapanch Nair  
Institute for Multiscale Simulation  
Friedrich-Alexander Universität Erlangen-Nürnberg  
Erlangen.

Thorsten Pöschel  
Institute for Multiscale Simulation  
Friedrich-Alexander Universität Erlangen-Nürnberg, Erlangen.

Abstract—Parts manufactured by additive manufacturing may suffer from anisotropic mechanical properties [1] caused by processing conditions or the geometric properties of the applied powder bed [2]. A reason for this is the formation of pores during processing which leads to a reduced bulk density. In order to optimize the material behavior of such parts a quantitative understanding of the interplay of parameters and material properties is necessary [3], [4]. In this work a Smoothed Particle Hydrodynamics (SPH) code is used that is capable of simulating the laser induced heat flow in a body. Therefore, the heat conduction is solved together with a volumetric source term by using De-Beer-Lamber’s law. The accuracy of the implemented model is then validated against analytical solutions in a cuboid for a static as well as a moving laser heat source. The temporal evolution of the temperature field in a cuboid is then compared by using a constant and a evolving beam radius depending on the depth of focus. Finally, the temperature field evolution in a convex shaped hemisphere is compared to the temperature field underneath a plane surface.

I. INTRODUCTION

Almost all metallic materials processed by Selective Laser Melting (SLM) suffer from pore formation. This leads to anisotropic mechanical material properties and as a result quality criteria, e.g. in the aerospace industry, can not be achieved. Because experimental studies on this are cost and time expensive, simulations present an efficient way to study the microstructure formation. In literature Smoothed Particle Hydrodynamics (SPH) has been widely applied to heat transfer problems [5], [6]. SPH being an updated Lagrangian simulation method handles scalar transport accurately. Hence, heat transfer can be simulated with large gradients in conductivity[7]. Previous studies in SPH using a laser model are made by neglecting the laser attenuation and assuming a surface heat source which is sufficient to model laser welding processes [8]–[10]. However, due to long processing times in AM processes heat conduction can not be neglected and the shape of the propagation front of the laser into the material must be considered. In this work a Gaussian laser model which includes an attenuation along the depth of focus according to De-Beer-Lamber’s law is implemented and validated against analytical results. Dirichlet boundary conditions are used to mimic a semi-infinite body as assumed by the analytical solution. Thereafter, an irradiated cuboid with Neumann boundary condition on each site is simulated. The influence of a varying beam radius along the propagation direction of the laser field is investigated. Finally, the temperature field within a convex shaped hemisphere is compared to the temperature field underneath a plane surface.

II. SPH FORMULATION

A. Governing Equations

The effect of laser heating is introduced by adding a volumetric source term $Q$ to the heat conduction equation given in eq. (1), which is modeled as a scalar transport equation.

$$\frac{\partial T}{\partial t} = \frac{1}{\rho} \nabla \cdot (\kappa \nabla T) + Q$$ (1)

Here, $c_p$ is the heat capacity at constant pressure, $\kappa$ is the thermal conductivity, $\frac{\partial T}{\partial t}$ is the temperature change per time and $\rho$ is the density of the material.

The laser beam is approximated by a transversal Gaussian intensity profile with a 1/e radius while propagating in the $z$-direction as shown in eq. (2) [11]. Here, $r$ is the radial distance to the center of the beam spot, $I_0$ is the initial intensity and $P$ is the power of the laser.

$$Q(r, z) = \frac{\alpha P}{\pi w(z)^2} \exp(-\alpha z) \exp\left(-\frac{r^2}{w(z)^2}\right)$$ (2)

Furthermore, the intensity decays along the propagation direction according Lambert-Beers law, where $\alpha$ is the absorptivity of the material. The beam radius $w(z)$ depends on the depth of focus $z$, given by

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2},$$ (3)

where $w_0$ and $z_R$ are the beam waist radius and the Rayleigh length, respectively [12]. The Rayleigh length of a laser beam is the distance from the beam waist radius $w_0$ where it increased by a factor of $\sqrt{2}$ and hence the cross-sectional area of the beam has doubled. It determines the depth of focus as
In order to validate the accuracy of the implemented heat conduction equation together with the laser source term the heating of a cuboid with length and width equal to \( L = W = 5 \text{m} \), height \( H = 0.5 \text{m} \), and the laser beam an inverse distance weighting of all surface particles could result from taking derivatives of derivatives.

\[ C \frac{dT_a}{dt} = \sum_b \frac{m_b}{\rho_a \rho_b} (k_a + k_b)(T_a - T_b) F_{ab} + Q \]  

(5)

In the discretized heat conduction equation the radial derivative of the kernel function is denoted as \( F_{ab} \), \( m \) is the mass and \( \rho \) is the density of a particle. In this work the Wendland C2 kernel [14] is used to interpolate the thermal contribution of the neighboring particles on the central particle.

In order to calculate the distance of an SPH particle to the surface, where the laser enters the material, a two-step algorithm proposed by Marrone, Colagrossi, Le Touzé, et al. [15] is used. The first step takes advantage of the properties of the renormalization matrix \( B [16] \). Therefore, the minimum eigenvalue of \( B^{-1} \) [17] is calculated from eq. (6) in order to find particles in the vicinity of the surface. This step is only used to minimize the number of particles that must be checked for being surface particles.

\[ B^{-1}(r_a) = \sum_b \frac{m_b}{\rho_b} \nabla W_b(r_a) \otimes (r_b - r_a) \]  

(6)

In the second step the corresponding particles with a minimum eigenvalue smaller than 0.75 are tested to be a surface particle. Therefore, the normal vector of each particle near the free surface must be computed which can be done by again using the renormalization matrix as

\[ n_a = \frac{-B(r_a) \sum_b m_b/\rho_b \nabla W_b(r_a)}{|-B(r_a) \sum_b m_b/\rho_b \nabla W_b(r_a)|} \]  

(7)

With the normal vector a scan region as shown in fig. 2 can be defined. By checking each neighboring particle \( b \) of a central particle \( a \) for the conditions given by

\[ \forall b \in \mathbb{N} \left[ |r_{ba}| \geq \sqrt{2}h, |r_{ba}| < h \right] \]

\[ \forall b \in \mathbb{N} \left[ |r_{ba}| < \sqrt{4}h, \arccos \left( \frac{n \cdot r_{ba}}{|r_{ba}|} \right) < \frac{\pi}{4} \right] \]  

(8)

it can be identified to be inside or outside the scan regions \( S1 \) and \( S2 \). Here, \( T \) is a point in a \( h = 1.33\Delta x \) distance in normal direction of the position of particle \( a \), where \( \Delta x \) is the particle diameter, and \( r_{ba} = r_b - r_a \) is the distance between particle \( a \) and \( b \). The detected surface particles in case of an irregular shaped particle are shown in fig. 3. In order to calculate the surface distance of a particle in dependence of the direction of the laser all particles are projected into an orthogonal plane to the origin of the laser beam. In order to approximate the distance of the surface particles to the beam an inverse distance weighting of all surface particles lying within a \( 3\Delta x \) radius around the investigated particle in the projected plane is considered. The surface distance is computed as the difference of the weighted surface particle to plane and the particle to plane distance.

C. Time update

The heat conduction (5) as well as the laser position are updated using explicit Euler integration.

III. VALIDATION AND RESULTS

In order to validate the accuracy of the implemented heat conduction equation together with the laser source term the heating of a cuboid with length and width equal to 0.1\( E-5 \) m,
as well as a height of $5.0 \times 10^{-6}$ m is investigated. The SPH particle diameter is set to $\Delta x = 3.23 \times 10^{-6}$ m, $\Delta x = 2.44 \times 10^{-6}$ m and $\Delta x = 1.96 \times 10^{-6}$ m resulting in a spatial resolution of $31 \times 31 \times 16$, $41 \times 41 \times 21$ or $51 \times 51 \times 26$ uniformly distributed particles within the domain. The analytical solution \cite{11} is given by

$$
\Theta(\chi, \xi, \tau, \nu, \gamma) = \frac{P}{2\sqrt{\pi}} \int_{\tau_0}^{\tau + \xi} \left[ \exp \left( \frac{\chi^2 + \xi^2}{\tau' + 1} \right) \right] \times \exp \left( -\frac{(\chi - \nu (\tau - \tau'))^2 + \xi^2}{\tau' + 1} \right) \times \left[ \exp (\zeta) \text{erf} \gamma \sqrt{\tau'} + \frac{\zeta}{2\sqrt{\pi}^{\tau'}} \right] + \exp (-\zeta) \text{erf} \gamma \sqrt{\tau'} - \frac{\zeta}{2\sqrt{\pi}^{\tau'}} \right]
$$

where $\chi = x/w_0$, $\xi = y/w_0$, $\zeta = \alpha z$, $\tau = 4Dt/w_0$, $\nu = v_{wu}/4D$ and $\gamma = \alpha w_0/2$. The laser velocity is denoted as $v$ and the thermal diffusivity is defined as $D = k/\rho c_p$.

### A. Static laser heat source

The initial SPH particle positioning for a resolution of $31 \times 31 \times 16$ is shown in fig. 4. Hollow circles represent solid particles and filled circles denote wall particles in order to satisfy Dirichlet BC. The number of wall particle layers is determined by the smoothing length and varies between three and six layers for $h = 1.5$ and $h = 2.75$. The laser radius is set to $w_0 = 1 \times 10^{-3}$ m for all simulations in this work and is constant $w(z) = w_0$ along the laser direction $(0, 0, -1)$ for all validation cases. Furthermore, the wavelength is set to $1.06 \times 10^{-5}$ for all simulations and corresponds to that of a $CO_2$-laser. The power of the laser is $P = 0.01$ W. The material properties are assumed to be constant and shown in table I. The timestep is set to $\Delta t = 5 \times 10^{-7}$ s.

The relative error of the SPH temperature to the analytical solution is calculated by

$$
err = 100 \frac{1}{N} \sum_{a=1}^{N} \frac{|T_a - T_{anl}|}{|T_{anl}|},
$$

where $N$ is the total number of SPH particles at a specific distance to the surface within $3w_0$ radius from the center of the laser beam. The dimensionless temperature field for $\chi = 0$ and $-3 \leq \xi \leq 3$ at the surface $\zeta = 0$ for a resolution of $51 \times 51 \times 26$ SPH particles is shown in fig. 5. Here, the smoothing length is $h = 1.5$. Solid lines and hollow circles represent the analytical solution and SPH particles, respectively. The longer the laser lasts on the cuboid, the higher the temperature rises.

Furthermore, the peak temperature occurs in the center of the laser beam, which is $\Theta = 1.02 \times 10^{-2}$ at $\tau = 0.2$ and $\Theta = 3.83 \times 10^{-2}$ at $\tau = 1.0$. In contrast to the analytical solution the SPH simulation predicts a higher temperature.

Towards the boundary of the laser irradiated domain the temperature decreases exponentially. The temporal evolution of the mean average error (eq. (10)) for the three investigated spatial SPH resolutions and a smoothing length of $h = 2.5$ are shown in fig. 6. As the time proceeds the error increases until it reached its maximum at $\tau = 2.0$, $\tau = 1.8$ and $\tau = 1.4$ for a resolution of $31 \times 31 \times 16$, $41 \times 41 \times 21$ or $51 \times 51 \times 26$. The corresponding errors are $17.1 \%$, $13.7 \%$ and $11.6 \%$. A reason for the increasing error with time could be a reduced thermal flow due to heat conduction compared to the analytical solution which causes an accumulation of heat at the surface and hence a higher temperature. Due to the heat input from the laser and the distribution among neighboring particles a steady state will be reached after a certain time. That is why the temperature change per time of particles

### TABLE I: Thermodynamic and optical properties of the SPH particles.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.10</td>
<td>W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>1.00 $\times$ 10$^3$</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.00 $\times$ 10$^5$</td>
<td>m$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.00 $\times$ 10$^3$</td>
<td>kg m$^{-3}$</td>
</tr>
</tbody>
</table>

(a) Cross-sectional x-y plane of the cuboid.

(b) Cross-sectional x-z plane of the cuboid.

Fig. 4: Initial particle positioning for the validation of the heat conduction equation with laser source term. Hollow circles represent solid particles and filled circles denote wall particles in order to satisfy Dirichlet BC.

Fig. 5: Radial temperature field of a cuboid on the surface at $\tau = 1.00$ obtained from a static laser with constant beam radius. Hollow circles denote the SPH temperatures whereas the solid lines show the temperatures obtained the analytical solution.
affected by the laser reduces and the error decreases after the respective peak was reached. In fig. 7 the temporal evolution of the error for different smoothing length $h$ and a spatial resolution of $51 \times 51 \times 26$ is shown. The lowest error occurs for a smoothing length of $h = 1.5$. Here, the maximum error is $6.2\%$ at $\tau = 1.0$. With increasing smoothing length the error increases towards a maximum error of $12.9\%$ at $\tau = 1.4$ for $h = 2.75$. In order to investigate the accuracy of the attenuation of the laser beam with increasing penetration depth the dimensionless temperature at the center of the beam as well as at a radial distance of $\xi = 0.78$ and $\xi = 1.57$ is measured. Fig. 8 shows the temperature field at $\tau = 1.0$ along direction of the laser into the cuboid which is compared against the analytical solution. With increasing surface distance the temperature decreases and is in good agreement with the analytical prediction. Only for particles in the vicinity of the surface the temperature deviation increases. This absolute temperature difference to the analytical solution decreases with increasing radial distance. The corresponding relative error to the analytical solution for each of the investigated particles in fig 8 is shown in fig. 9. With increasing distance to the surface and the beam center the error decreases until it approaches its lowest value for a surface distance between $\zeta = 0.27$ and $\zeta = 0.67$. The error takes values between $5.08\%$ and $0.21\%$ for the $\xi = 0$ and $\xi = 0.78$.

**B. Moving laser heat source**

In this subsection the temperature rise due to a moving laser heat source with a dimensionless velocity $\nu = 2.5$, $\nu = 7.5$, $\nu = 25.0$ is compared against the analytical solution at different times. Fig. 10 shows the temperature at $\tau = 0.2$. An increase in the laser movement speed along the $\xi$ direction leads to a shifted Gaussian distribution resulting in a lower peak temperature for high velocities. For a pulse
length $\tau = 1.0$ and a velocity $\nu = 25.0$, the temperature deviation from the analytical solution increasing along the moving direction of the laser. The reason for this could be an uneven movement of the laser and hence, a disturbed heat input into the body due to a too large timestep. The radial temperature at $\tau = 1.0$ is shown in fig. 11

Fig. 11: Radial temperature fields caused by a moving laser at $\tau = 1.0$ for different velocities.

C. Static laser heat source for different beam radii

In this subsection the influence of a varying radius $w(z)$ along the propagation direction of the laser on the temperature field within a cuboid and a hemisphere analyzed. In fig. 12 the dimensionless radial temperatures at $\tau = 1.00$ for a constant ($w = w_0$) and a variable beam radius ($w = w(z)$) in a cuboid are shown. The temperatures obtained from $w(z)$ are higher than those from a constant radius along the direction of the beam. The difference may be caused by a lower temperature gradient in radial direction with increasing depth of focus of the laser by distributing the inserted energy among a larger area. Additionally, with increasing distance to the surface the temperatures caused by a variable beam radius differ as shown in fig. 13. Near the surface the temperature is higher than with constant $w_0$, while the temperatures with a larger distance to the surface are lower than with constant beam radius $w_0$. In additive manufacturing the surface of powder beds have an irregular shape due to the the size distribution as well as the irregular spherical shaped granules. For this purpose the heating process of a hemisphere with an underlying cuboid is compared to the heating of a cuboid with a flat surface. The geometry of the hemisphere with radius $\sqrt{\xi^2 + \chi^2} = 5$ is
shown in Fig. 14. The dimensions of the underlying cuboid are $\xi \times \chi \times \zeta = (12 \times 12 \times 0.2)$. The dimensionless temperature of the particle at $\chi = \xi = \zeta = 0$ for different times are shown in Fig. 15 for the hemisphere and the cuboid. The peak surface temperatures obtained by a varying beam radius are higher than those obtained by a constant beam radius. But also, the temperature of the hemisphere at the beam center is higher than that of the cuboid.

The temperature along the propagation direction of the laser at different radial distances from the beam center is shown in Fig. 16. It can be seen that the temperature along the propagation direction of the beam strongly differs depending on the chosen beam radius. For a varying beam radius the temperature decays more than for a constant beam radius. The smaller the radial distance from the beam center the higher is the temperature decrease. A reason for this is the higher area affected by the laser with increasing depth of focus in which the same amount of energy is distributed.

Finally, the axial temperatures of the square and the hemisphere are at different radial distances are compared for a varying beam radius in Fig. 17. It can be seen that the hemisphere acquires higher temperatures as the cuboid. This effect is caused by a reduced heat conduction due to the lack of material due to the spherical shape compared to the cuboid.

**IV. Conclusion**

The shape of a laser as well as the material properties influence the heat transfer in a body. This affects the melt flow in case of laser beam melting which leads to pores in the material. In order to optimize the processing parameters...
in laser beam melting a laser model was validated against analytical solutions for a static and a moving laser. The model is then used to heat the surface of a cuboid and a hemisphere with a constant as well as a varying beam radius in dependence of the depth of focus. It could be seen that the temperature field strongly depends on the chosen radius of the laser beam. By using a varying beam radius along the propagation direction of the laser beam the temperature decays faster compared to the simulations using a constant beam radius. Moreover, it is observed that the temperature field of the hemisphere acquires higher temperatures than those of the cuboid. A reason for this is the limited heat distribution among the material due to the convex shape. Further studies are required to investigate the relationship between beam shape, absorptivity of the material, beam intensity and velocity together with a phase change and surface tension model on the melt flow of a powder bed in AM. This will be addressed in a future study.

REFERENCES


