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Rolling friction of a viscous sphere on a hard plane

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Abstract. – A first-principle continuum-mechanics expression for the rolling friction coefficient is obtained for the rolling motion of a viscoelastic sphere on a hard plane. It relates the friction coefficient to the viscous and elastic constants of the sphere material. The relation obtained refers to the case when the deformation of the sphere ξ is small, the velocity of the sphere V is much less than the speed of sound in the material and when the characteristic time ξ/V is much larger than the dissipative relaxation times of the viscoelastic material. To our knowledge this is the first "first-principle" expression of the rolling friction coefficient which does not contain empirical parameters.

Rolling friction is one of the basic phenomena man encounters in his everyday life since antediluvian times when the wheel was invented. The phenomenon of rolling friction has been interesting to scientists for a long time. Scientific publications on this subject range back to (at least) 1785 when Vince described systematic experiments to determine the nature of friction laws [1], and important scientists dealt with this problem, among them Reynolds [2]. The rolling friction is of great importance in engineering and science. From the speed of landslides Huang and Wang [3] argued that the rolling friction plays an important role even in geological processes, for a theoretical consideration see [4]. For its major importance the phenomenon has been studied intensively by engineers and physicists (*e.g.*, [5]), however, surprisingly little is known about its basic mechanisms. To our knowledge there is still no "first-principle" expression for the rolling friction coefficient available which relates this coefficient only to the material constants of the rolling body and does not contain empirical parameters.

It has been shown that surface effects like adhesion [6], electrostatic interaction [7], and other surface properties [8] might influence the value of the rolling friction coefficient. Theoretically this problem was studied in ref. [9] where the authors propose a model of a surface with asperities to mimic friction (see also [10]). In other studies [11], [12] it was argued that for

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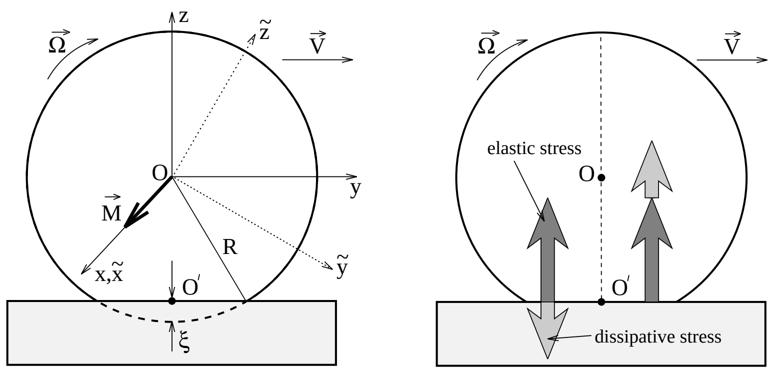

Fig. 1. – Sketch of the rolling sphere, the right figure shows the appearance of the friction momentum: in the front the elastic stress is enhanced by the dissipative stress, while in the rear it is diminished.

viscoelastic materials "the rolling friction is due very little to surface interactions: the major part is due to deformation losses within the bulk of the material" [11]. Based on this concept the rolling friction coefficient was calculated in [11] where the deformation in the bulk was assumed to be completely plastic; then an empirical coefficient was introduced to account for the retarded recover of the material.

In the present letter we also consider the rolling friction as a phenomenon appearing due to viscous processes in the bulk. We assume that energy losses due to surface effects may be neglected, compared to the viscous dissipation in the bulk. Thus, as in [11], [12], we attribute the effect of rolling friction to viscous dissipation in the material due to time-dependent deformation. We assume that the only role of surface forces is to keep the rolling body from sliding. We use a quasi-static approach [13] and obtain the rolling friction coefficient.

Figure 1 shows a sketch of the sphere of radius R and mass m rolling with angular velocity Ω which is related to its linear velocity V as $V = \Omega R$. The center of the coordinate system $OXYZ$ coincides with the center of the sphere (moving coordinate system), so that the plane is normal to the Z-axis and located at $z_p < 0$. Within our model we assume that the surface is much harder than the sphere, hence, the surface remains flat in the contact area while the sphere is deformed [14]. Thus, the contact area is a circle of radius a. The compression $\xi = R - |z_p|$ of the sphere causes the displacement field in the bulk of the material $\mathbf{u}(\mathbf{r}, t)$, where $\mathbf{r}$ gives the location of the point within the sphere at time t. The displacement field gives rise to a (tensorial) field of deformation $\boldsymbol{\nabla} \circ \mathbf{u}$, and thus to the elastic part of the stress tensor $\hat{\sigma}_{\rm el}$ [15]

$$\hat{\sigma}_{\rm el} = E_1 \left[\frac{1}{2} \left\{ \boldsymbol{\nabla} \circ \mathbf{u} + \mathbf{u} \circ \boldsymbol{\nabla} \right\} - \frac{1}{3} \hat{I}\, \boldsymbol{\nabla} \cdot \mathbf{u} \right] + E_2\, \hat{I}\, \boldsymbol{\nabla} \cdot \mathbf{u} \,. \tag{1}$$

Here, $\hat{I}$ is the unit tensor and $E_1 = \frac{Y}{1+\nu}$ and $E_2 = \frac{Y}{3(1-2\nu)}$ denote the elastic material constants with Y and ν being the Young modulus and the Poisson ratio, respectively. Since the deformation depends on time via the time-dependent field $\mathbf{u}(\mathbf{r}, t)$, the dissipative part of the stress tensor reads

$$\hat{\sigma}_{\rm dis} = \eta_1 \left[\frac{1}{2} \left\{ \boldsymbol{\nabla} \circ \dot{\mathbf{u}} + \dot{\mathbf{u}} \circ \boldsymbol{\nabla} \right\} - \frac{1}{3} \hat{I}\, \boldsymbol{\nabla} \cdot \dot{\mathbf{u}} \right] + \eta_2\, \hat{I}\, \boldsymbol{\nabla} \cdot \dot{\mathbf{u}} \,, \tag{2}$$

where $\dot{\mathbf{u}}$ denotes the time derivative of the displacement field and η_1 and η_2 are the viscous constants of the sphere material [15]. The total stress $\hat{\sigma}$ is a sum of both parts, $\hat{\sigma} = \hat{\sigma}_{\rm el} + \hat{\sigma}_{\rm dis}$.

The force normal to the plane which acts on the sphere is caused by its own weight mg. It reads

$$\mathbf{F} = \int_{S_a} \hat{\sigma} \cdot \mathbf{n} \, \mathrm{d}s = \mathbf{n} \int\int \mathrm{d}x\mathrm{d}y \; (\sigma_{\mathrm{el}}^{zz} + \sigma_{\mathrm{dis}}^{zz}) = \mathbf{n} F^N \,, \tag{3}$$

where $\mathbf{n}$ is the (positively directed) unit normal to the plane, $\sigma_{\mathrm{el}}^{zz}$ and $\sigma_{\mathrm{dis}}^{zz}$ are the corresponding components of the stress tensors. The integration is to be performed over the contact circle S_a on the plane $z = z_{\mathrm{p}}$. Correspondingly, the torque acting on the sphere with respect to the point O' at rest (no slipping conditions) is given by

$$\mathbf{M} = \int_{S_a} (\mathbf{r} - \mathbf{r}_{O'}) \times (\hat{\sigma} \cdot \mathbf{n}) \, \mathrm{d}s \,, \tag{4}$$

where $\mathbf{r}_{O'} = (0, 0, z_{\mathrm{p}})$. The torque $\mathbf{M}$ thus characterizes the rolling friction.

Let the sphere move in the y-direction, $\mathbf{V} = (0, V, 0)$, then the angular velocity and the friction torque are directed along the x-axis, *i.e.* $\Omega = (-\Omega, 0, 0)$ and $\mathbf{M} = (M, 0, 0)$ (see fig. 1). For this geometry from eq. (4) follows

$$M = \int\int \mathrm{d}x\mathrm{d}y \; y \, \sigma^{zz} \, (x, y, z = z_{\mathrm{p}}) \; . \tag{5}$$

The integration in eq. (5) again is performed over the contact circle.

To perform the calculation of the friction torque one needs the displacement field $\mathbf{u}(\mathbf{r}, t)$ and the field of the displacement velocity $\dot{\mathbf{u}}(\mathbf{r}, t)$. Generally, to find these quantities is a rather complicated problem. Nevertheless, one can use a quasi-static approximation, provided that the displacement velocities in the bulk are much smaller than the speed of sound in the sphere material and provided the characteristic time of the process of interest, estimated by $\tau = \xi/V$, is much larger than the dissipative relaxation times of the material (see [13] for details). This allows to employ for the displacement field $\mathbf{u}$ the results of the static problem, *i.e.* of the Hertz contact problem which refers to a "slow" elastic collision of two spheres [16]. In the case considered here one of the spheres (the plane) should have infinite radius. Let $\mathbf{u}_{\mathrm{el}}(\mathbf{r})$ be the solution of this (static) contact problem. It corresponds to the displacement field in a sphere under the compression ξ (see fig. 1) if it were at rest. In the quasi-static approximation the distribution $\mathbf{u}_{\mathrm{el}}(\mathbf{r})$ persists (*i.e.* it does not change with time) in the coordinate system $OXYZ$, moving with the velocity V (with no rotation). In a body-fixed coordinate system $O\tilde{X}\tilde{Y}\tilde{Z}$ (see fig. 1), which rotates with the angular velocity Ω and coincides at some time instant with $OXYZ$ the displacements distribution is the same as in $OXYZ$ (*i.e.* $\mathbf{u}_{\mathrm{el}}(\mathbf{r})$), while the displacement velocity distribution follows from the kinematic equation

$$\dot{\mathbf{u}}(\mathbf{r}) = (\boldsymbol{\Omega} \cdot \mathbf{r} \times \boldsymbol{\nabla}) \mathbf{u}_{\mathrm{el}}(\mathbf{r}) = -\Omega \, (y \, \partial_z - z \, \partial_y) \; \mathbf{u}_{\mathrm{el}}(\mathbf{r}) \tag{6}$$

which relates both coordinate systems; here $\partial_x = \frac{\partial}{\partial x}$, $\partial_y = \frac{\partial}{\partial y}$ and $\partial_z = \frac{\partial}{\partial z}$. The elastic part of the stress tensor is then obtained by substituting $\mathbf{u}_{\mathrm{el}}(\mathbf{r})$ into eq. (1). This part of the stress tensor corresponds to the static case when no torque acts on the sphere. Therefore, in the quasi-static approximation only the dissipative part of the stress tensor $\sigma_{\mathrm{dis}}^{zz}$ should be used in eq. (5) for the friction torque. Using the kinematic equation (6) and definitions (1) and (2) one finds for the dissipative part

$$\begin{aligned} \sigma_{\mathrm{dis}}^{zz} = & -\Omega \left\{ (y \, \partial_z - z \, \partial_y) \, \sigma_{\mathrm{el}}^{zz} \, (E_1 \leftrightarrow \eta_1; E_2 \leftrightarrow \eta_2) + \right. \\ & \left. +2\sigma_{\mathrm{el}}^{yz} (E_1 \leftrightarrow (\eta_2 - \eta_1/3); E_2 \leftrightarrow \eta_2) - (2\eta_2 + \eta_1/3) \partial_y u_{\mathrm{el}}^z \right\} \,, \end{aligned} \tag{7}$$

where $\mathbf{u}_{\rm el} = (u^x_{\rm el}, u^y_{\rm el}, u^z_{\rm el})$ and $E_1 \leftrightarrow \eta_1$, $E_2 \leftrightarrow \eta_2$, etc. in the arguments of the *elastic* stress tensor denote that the elastic constants E_1, E_2, etc. in eq. (1) should be replaced by the dissipative constants η_1, η_2, etc.

For the case of small compression $\xi/R \ll 1$ which is addressed here we estimate the magnitude of different terms in eq. (7) taken at the plane $z = z_p \approx -R$, since these values of the stress tensor determine the friction torque (see eq. (5)). For the diagonal component of the stress tensor one has the known solution for the Hertz contact problem [16], [15]

$$\begin{aligned}\sigma^{zz}_{\rm el}(x,y,z_{\rm p}) &= E_1\partial_z u^z_{\rm el} + (E_2 - E_1/3)\left(\partial_x u^x_{\rm el} + \partial_y u^y_{\rm el} + \partial_z u^z_{\rm el}\right)\\ &= \frac{3}{2}\frac{F^N_{\rm el}}{\pi ab}\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}\,,\end{aligned}\tag{8}$$

where $a = b = \sqrt{2\xi R}$ is the radius of the contact circle and $F^N_{\rm el}$ is the total elastic force, acting by the surface (in normal direction) on the sphere:

$$F^N_{\rm el}(\xi) = \frac{2}{3}\frac{Y}{(1-\nu^2)}R^{1/2}\,\xi^{3/2}\,. \tag{9}$$

From eq. (9) follows $\sigma^{zz}_{\rm el} \sim (\xi/R)^{1/2}$ at $z = z_{\rm p}$. Applying geometric consideration of the deformation of the sphere one gets $u^x_{\rm el} \sim u^y_{\rm el} \sim \xi^{3/2}/R^{1/2}$, and $u^z_{\rm el} \sim \xi$. The estimates $\partial_x \sim \partial_y \sim 1/a \sim (\xi R)^{-1/2}$ and $\partial_z \sim (\xi R)^{-1/2}$ may be obtained too. The expression for ∂_z follows from the estimates for $u^z_{\rm el}$, $\sigma^{zz}_{\rm el}$ and the definition of $\sigma^{zz}_{\rm el}$. Finally we find for the deformation

$$\begin{aligned}\partial_x u^x_{\rm el} \sim \partial_y u^x_{\rm el} \sim \partial_z u^x_{\rm el} &\sim \xi/R \ll (\xi/R)^{1/2}\,,\\ \partial_x u^y_{\rm el} \sim \partial_y u^y_{\rm el} \sim \partial_z u^y_{\rm el} &\sim \xi/R \ll (\xi/R)^{1/2}\,,\\ \partial_x u^z_{\rm el} \sim \partial_y u^z_{\rm el} \sim \partial_z u^z_{\rm el} &\sim (\xi/R)^{1/2}\,.\end{aligned}\tag{10}$$

Equations (10) further yield $\sigma^{yz}_{\rm el} \sim (\xi/R)^{1/2}$. Using these estimates one can calculate the relative magnitude of the terms in eq. (7) and then compare their contribution to the rolling friction torque M. After these calculations one arrives at

$$M = -\Omega R\int\!\!\int {\rm d}x{\rm d}y\; y\partial_y\,\sigma^{zz}_{\rm el}\left(E_1\leftrightarrow\eta_1; E_2\leftrightarrow\eta_2\right)\,, \tag{11}$$

where all other terms which are smaller by a factor $(\xi/R)^{1/2}$ are omitted. Integrating by parts the right-hand side of eq. (11) and taking into account that the stress tensor vanishes on the boundary of the contact circle one finally finds

$$M = \Omega R\left\{\int\!\!\int {\rm d}x{\rm d}y\;\sigma^{zz}_{\rm el}\left(E_1\leftrightarrow\eta_1; E_2\leftrightarrow\eta_2\right)\right\}\,. \tag{12}$$

The expression in the curly brackets in eq. (12) would coincide with the normal elastic force $F^N_{\rm el}$ (see eq. (3)) if it would contain the elastic constants E_1 and E_2 instead of the dissipative constants η_1 and η_2. To perform the integration in eq. (12) we apply the rescaling procedure proposed in [13]. The coordinates are transformed due to $x = \alpha x'$, $y = \alpha y'$ and $z = z'$ with

$$\alpha = \left(\frac{\eta_2 - \frac{1}{3}\eta_1}{\eta_2 + \frac{2}{3}\eta_1}\right)\left(\frac{E_2 + \frac{2}{3}E_1}{E_2 - \frac{1}{3}E_1}\right)\,, \tag{13}$$

so that the radius of the contact area in new coordinates is $a = b = \alpha a' = \alpha b'$. One obtains

$$\begin{aligned}
\sigma_{\rm el}^{zz}\left(E_1 \leftrightarrow \eta_1; E_2 \leftrightarrow \eta_2\right) &= \beta\left(\eta_1 \frac{\partial u_{\rm el}^z}{\partial z} + \left(\eta_2 - \frac{\eta_1}{3}\right)\left(\frac{\partial u_{\rm el}^x}{\partial x} + \frac{\partial u_{\rm el}^y}{\partial y} + \frac{\partial u_{\rm el}^z}{\partial z}\right)\right) \\
&= \beta\left(E_1 \frac{\partial u_{\rm el}^z}{\partial z'} + \left(E_2 - \frac{E_1}{3}\right)\left(\frac{\partial u_{\rm el}^x}{\partial x'} + \frac{\partial u_{\rm el}^y}{\partial y'} + \frac{\partial u_{\rm el}^z}{\partial z'}\right)\right) \\
&= \beta \frac{3}{2}\frac{F_{\rm el}^N}{\pi a' b'}\sqrt{1 - \frac{x'^2}{a'^2} - \frac{y'^2}{b'^2}} \\
&= \beta\alpha^2\, \frac{3}{2}\frac{F_{\rm el}^N}{\pi a b}\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}\,,
\end{aligned} \tag{14}$$

where $\beta = \frac{1}{\alpha}\frac{\eta_2 - \frac{1}{3}\eta_1}{E_2 - \frac{1}{3}E_1}$. As follows from eq. (14) after the appropriate rescaling, the desired expression for the stress tensor with the "replaced" constants may be written in the same way as in (9). Integration in eq. (12) yields the final result for the torque of the rolling friction in the quasi-static regime

$$M = k_{\rm rol}\varOmega R F^N\,. \tag{15}$$

It may be shown that in the quasi-static regime the normal force F^N which acts on the sphere coincides with $F_{\rm el}^N$ of the static problem. Figure 1 explains the appearance the friction torque in the rolling motion of a viscous sphere on a hard plane [14]. The rolling friction coefficient finally reads [17]

$$k_{\rm rol} = \frac{1}{3}\,\frac{(3\eta_2 - \eta_1)^2}{(3\eta_2 + 2\eta_1)}\,\left[\frac{\left(1-\nu^2\right)\left(1-2\nu\right)}{Y\,\nu^2}\right]\,. \tag{16}$$

The latter expression relates the rolling friction coefficient to the elastic and viscous constants of the rolling body material. Note that eq. (16) does not contain $\varOmega$; thus for the case of a soft sphere rolling on a hard plane [14] the torque linearly increases with increasing velocity $V = \varOmega R$. Physically, this reflects the fact that dissipative forces in the bulk, which are responsible for rolling friction, scale linearly with the product $\varOmega R$ (see eqs. (2) and (6)).

It is interesting to note that the rolling friction coefficient $k_{\rm rol}$ coincides with the material constant A in the dissipative force $F_{\rm dis}^N = \rho\, A\,\sqrt{\xi}\dot{\xi}$, $\rho = \frac{Y}{(1-\nu^2)}\sqrt{R/2}$ for two colliding spheres (for derivation see [13]). Using this expression one can calculate the normal coefficient of restitution ϵ for two spheres which collide with relative velocity v [13]. This coefficient which measures the energy loss upon collision reads $1 - \epsilon = C_1\,A\,\rho^{2/5} v^{1/5} + \cdots$ $(C_1 = 1.15344)$ [18]. Therefore, one can find the rolling friction coefficient from the restitution coefficient of the same material. To get an estimate of $k_{\rm rol}$ we take $1 - \epsilon = 0.1$ for steel particles of mass $m = 10^{-2}\,\mathrm{kg}$, colliding with impact velocity $v = 1\,\mathrm{m/s}$ to obtain $\rho^{2/5} \sim 10^5\,(\mathrm{kg}^2/\mathrm{ms}^4)^{1/5}$ and $k_{\rm rol} \sim 10^{-6}\,\mathrm{s}$. To our knowledge for the case of a soft sphere rolling on a hard plane the dependence of the rolling friction coefficient on the rolling body velocity and radius has not been studied experimentally.

In conclusion, we found for the first time, to our knowledge, the first-principle expression which relates the rolling friction coefficient to the elastic and viscous properties of the material. Our calculations refer to the rolling motion of a viscoelastic sphere on a hard plane without slipping in the regime when the velocity of the sphere is much less than the speed of sound in the material and when the characteristic time of the process ξ/V is much larger than the dissipative relaxation times of the viscoelastic material.

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REFERENCES

[1] Vince S., *Philos. Trans. R. Soc. London*, **75** (1785) 165.

[2] Reynolds O., *Royal Society, Philos. Trans.*, **166** (1874) Pt. 1.

[3] Huang R. Q. and Wang S. T., in *Proc. Intern. Symp. Landslides, Lausanne, 1988*, edited by C. Bonnard, (1988), p. 187.

[4] Herrmann H. J., Mantica G. and Bessis D., *Phys. Rev. Lett.*, **65** (1990) 3223.

[5] Heathcote H. L., *Proc. Inst. Automot. Eng.*, **15** (1921) 569; Bentall R. H. and Johnson K. L., *Int. J. Mech. Sci.*, **6** (1967) 389; Czichos H., *Tribology* (Elsevier, Amsterdam) 1978; Barwell F. T., *Bearing Systems: Principles and Practice* (Oxford University Press, Oxford) 1979, pp. 312-315; Szeri A. Z., *Tribology-Friction, Lubrication, and Wear.* (Hemisphere, Washington) 1980, pp. 401-409; Dowson D., Taylor C. M., Childs T. H. C., Godet M. and Dalmaz G. (Editors), *Dissipative Processes in Tribology* (Elsevier, Amsterdam) 1994; Krause H. and Lehna H., *Wear*, **119** (1987) 153; Shpenkov G. P., *Friction Surface Phenomena* (Elsevier, Amsterdam) 1995; Rabinowicz E., *Friction and Waer of Materials* (Wiley, New York) 1965; zum Gahr K. R., *Microstructure and Wear of Material* (Elsevier, Amsterdam) 1987; Carter F. W., *Proc. R. Soc. London, Ser. A*, **112** (1926) 151; Domenéch, A., Domenéch, T. and Cebrián, J., *Am. J. Phys.*, **55** (1986) 231; Eldredge K. R. and Tabor D., *Proc. R. Soc. London, Ser. A*, **229** (1955) 180; Evseev D. G., Medvedev B. M., Grigoryan G. G. and Ermolin O. A., *Wear*, **167** (1993) 33; Li Y. F. and Seireg A., *J. Tribilogy-Transactions of the ASME*, **111** (1989) 386; Lingard S., *Wear*, **117** (1987) 109; Shaw D. E. and Wunderlich F. J., *Am. J. Phys.*, **52** (1984) 997; Vo P. H., *Proc. SPIE*, **1998** (1993) 141; Weltner K., *Am. J. Phys.*, **55** (1987) 937; Witters J. and Duymelink D., *Am. J. Phys.*, **54** (1986) 80.

[6] Barquins M., Maugis D., Blouet J. and Courtel R., *Wear*, **51** (1978) 375; Deryaguin B. V. and Toporov Y. P., *Prog. Surf. Sci.*, **45** (1994) 317; Kendall K., *Wear*, **33** (1975) 351; Roberts A. D. and Thomas A. G., *Wear*, **33** (1975) 45.

[7] Deryaguin B. V. and Smilga V. P., *Prog. Surf. Sci.*, **45** (1994) 108; Deryaguin B. V. and Smilga V. P., *Prog. Surf. Sci.*, **45** (1994) 296; Prashad H., *J. Tribilogy-Transactions of the ASME*, **110** (1988) 448.

[8] Chaplin R. L. and Chilson P. B., *Wear*, **107** (1986) 213; Song Q., *Am. J. Phys.*, **56** (1988) 1145; Chaplin R. L. and Miller M. G., *Am. J. Phys.*, **52** (1984) 1108.

[9] Fuller K. N. G. and Roberts A. D., *J. Phys. D*, **14** (1981) 221.

[10] Pöschel T. and Herrmann H. J., *Physica A*, **198** (1993) 441.

[11] Greenwood J. A., Minshall H. and Tabor D., *Proc. R. Soc. London, Ser. A*, **259** (1961) 480.

[12] Tabor D., *Proc. R. Soc. London, Ser. A*, **229** (1955) 198; Tabor D., *Philos. Mag.*, **43** (1952) 1055; Atack D. and Tabor D., *Proc. R. Soc. London, Ser. A*, **246** (1958) 539; Drutowski R. C., in *Proceedings of the Symposium on Friction an Wear, Detroit, 1957*, edited by R. Davies (Elsevier, Amsterdam) 1959, pp. 17.

[13] Brilliantov N. V., Spahn F., Hertzsch J.-M. and Pöschel T., *Phys. Rev. E*, **53** (1996) 5382.

[14] In most textbooks the opposite case of a hard sphere on a soft plane is considered, when the sphere creates a pit (or "step") on a surface. To escape from this pit a torque is required, thus rolling friction appears.

[15] Landau L. D. and Lifshitz E. M., *Theory of Elasticity* (Pergamon Press, London) 1959.

[16] Hertz H., *J. f. reine u. Angewandte Math.*, **92** (1882) 156.

[17] Usually the rolling friction coefficient is defined by $M = \mu F^N$, *i.e.* $\mu = k_{\mathrm{rol}} \Omega R$ and refers to a case, opposite to the one considered presently [14]. Writing eqs. (15), (16) we wish to stress the particular dependence of torque M on Ω and R for a viscous sphere rolling on a hard plane.

[18] Schwager T. and Pöschel T., *Phys. Rev. E*, **57** (1998) 650.