Comment on "Clustering of Granular Assemblies with Temperature Dependent Restitution under Keplerian Differential Rotation"

In a recent paper [1] Spahn, Schwarz, and Kurths report on the influence of the velocity dependent coefficient of restitution on the dynamics of a granular gas. It has been shown by several authors (e.g., [2]) that for the case of the constant coefficient of restitution there is a pressure instability which may result in cluster formation, provided that damping is high enough [3].

The authors [1] investigate now theoretically whether this cluster instability persists if the coefficient of restitution is a function of impact velocity. To this end, they use the equation of the collision $\ddot{\xi} \propto \xi^{3/2} + C\sqrt{\xi}\,\dot{\xi}$ with $\xi(0) = 0$, $\dot{\xi}(0) = v_{\rm imp}$, where ξ is the "deformation" of particles, C is a dissipative material constant, and $v_{\rm imp}$ is the impact velocity (for details, see [4]). From the numerical solution of this equation, one finds the coefficient of restitution $\epsilon(v_{\rm imp}) = -\dot{\xi}(t_c)/\dot{\xi}(0)$, where t_c is the duration of the collision. The authors solve the equation of motion numerically and fit

$$\dot{\epsilon}(v_{\rm imp}) = A/(v_{\rm imp} + v_*)^{\beta}, \qquad (1)$$

where A and β are fit parameters. v_* is adjusted to meet the condition $\epsilon(0) = 1$. For given material constants, the fit yields $A \approx 0.2 \ldots 0.4$ and $\beta \approx \frac{1}{4}$. Equation (1) is the foundation of all analytical results presented in [1].

Whereas (1) is certainly a good approximation for large $v_{\rm imp}$, it can be shown, however, that (1) is *not* the exact solution. Instead the solution reads

$$\epsilon(v_{\rm imp}) = 1 - \gamma_1 v_{\rm imp}^{1/5} + \gamma_2 v_{\rm imp}^{2/5} \mp \gamma_i v_{\rm imp}^{i/5}, \qquad (2)$$

$i = 3, 4, 5, \ldots$, where γ_1 and γ_2 are constants depending on material properties [5]. Figure 1 shows both Eqs. (1) and (2) over impact velocity. For large velocity, both expressions coincide; for small velocity, however, (1) overestimates the solution (2). Obviously, the limit $v_{\rm imp} \to 0$ is important for cluster formation.

In conclusion, we claim that the calculations in [1] concerning the tidal force driven clustering are qualitatively correct, since in this regime the lower velocity limit is perhaps less important and both expressions (1) and (2) coincide approximately. For the case of small impact velocity, however, both expressions differ as seen easily from Taylor expansion of Eqs. (1) and (2):

$$\epsilon_{(1)} \approx 1 - \frac{\beta}{A^{1/\beta}}\, v_{\rm imp}, \qquad \epsilon_{(2)} \approx 1 - \gamma_1 v_{\rm imp}^{\frac{1}{5}}, \qquad (3)$$

The deviation of (1) and (2) becomes important when the kinetic behavior is considered. The most obvious consequence of the functional form of $\epsilon(v_{\rm imp})$ is the decay of temperature of a force-free granular gas: Using

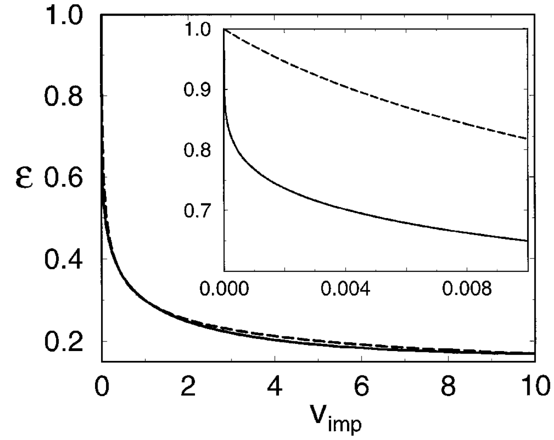

FIG. 1. Comparison of the fit (1) (dashed line) with the analytical solution (2) of the equation of motion (parameters are $A = 0.3$, $\beta = \frac{1}{4}$, $\gamma_1 = 1$, $\gamma_2 = 0.3$). For large impact velocity $v_{\rm imp}$, both curves agree. For the case of small $v_{\rm imp}$, however, (1) becomes incorrect.

(1), the authors find the implicit expression $t + t_0 \sim A^{-4}\{\epsilon(T)^2 + \ln[1 - \epsilon(T)^2]\}$ [1]. Using the analytical solution (2), one finds instead $T = T(0)\,(1 + t/t_0)^{-5/3}$. Hence, we believe that the results in [1] concerning the force-free case have to be reconsidered.

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